Prof Wavrik **Math 20B** **Winter 2006**

Midterm 2 -- ANSWERS

Part I - Short Answers

1. Give the **FORM** of the partial fractions decomposition for the following rational function, but do not evaluate the constants.

$$\frac{x^2+5}{x^2(x+1)(x^2+1)}=\frac{A}{x^2}+\frac{B}{x}+\frac{C}{x+1}+\frac{Dx+E}{x^2+1}$$

$$\frac{x^2+5}{x(x+1)(x^2+1)^2}=\frac{A}{x}+\frac{B}{x+1}+\frac{Cx+D}{(x^2+1)^2}+\frac{Ex+F}{x^2+1}$$

Given the partial fractions decomposition $\frac{x^2+2x-1}{x(x^2-1)}=\frac{1}{x}-\frac{1}{x+1}+\frac{1}{x-1}$ 2. Find $\int\frac{x^2+2x-1}{x(x^2-1)}\,dx=$ $\ln(x)+\ln(x-1)-\ln(x+1)+C$	Given the partial fractions decomposition $\frac{x^2+x+1}{x(x^2+1)}=\frac{1}{x}+\frac{1}{x^2+1}$ 2. Find $\int\frac{x^2+x+1}{x(x^2+1)}\,dx=\ln(x)+\arctan(x)+C$
3. Find $\int\frac{x^4+2x-1}{x(x^2-1)}\,dx=$ $\frac{1}{2}x^2+\ln(x)+\ln(x-1)-\ln(x+1)+C$	3. Find $\int\frac{x^4+2x^2+x+1}{x(x^2+1)}\,dx=$ $\frac{1}{2}x^2+\ln(x)+\arctan(x)+C$
4. Using trigonometric substitution x = T(t) (where T is one of sin, cos, tan, sec, csc, cotan), we can change $\int\sqrt{1-x^2}\,dx$ to one of the following integrals. Indicate which: a. $\int\tan^2(t)\sec(t)\,dt$ [✓ b.] $\int\cos^2(t)\,dt$ c. $\int dt$ d. None of the preceding	4. Using trigonometric substitution x = T(t) (where T is one of sin, cos, tan, sec, csc, cotan), we can change $\int\sqrt{1+x^2}\,dx$ to one of the following integrals. Indicate which: [✓ a.] $\int\sec^3(t)\,dt$ b. $\int\cos^2(t)\,dt$ c. $\int\sec(t)\,dt$ d. None of the preceding

$\int\frac{x^2}{1+x^2}\,dx=x-\arctan(x)+C$	$\int x\sqrt{x+1}\,dx=\frac{2}{5}(x+1)^{(5/2)}-\frac{2}{3}(x+1)^{(3/2)}+C$
$\int\ln(x)\,dx=x\ln(x)-x+C$	$\int\sin^2(x)\,dx=-\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x+C$
$\int x^2e^x\,dx=x^2e^x-2xe^x+2e^x+C$	$\int\frac{1}{x^2+x+1}\,dx=\frac{2}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}(2x+1)\sqrt{3}\right)+C$

Part II - Evaluating Integrals