Algebra Qual Prep: Summer, 2008.
Timed Ring Theory Problems

August 13, 2008

Unless otherwise stated, R is a ring with unity.

1. Suppose P is an ideal in a commutative ring R and that R/P is a finite integral domain. Prove P is a maximal ideal.
2. Define the following terms: irreducible element, prime element, prime ideal. Prove that in an integral domain a prime element is irreducible. Prove furthermore, that in a PID the converse holds. Now prove that in a PID all nonzero prime ideals are maximal.
3. Show that if a monic polynomial in $\mathbb{Z}[x]$ has a rational root then it has an integer root. State and prove Eisenstein's criterion of irreducibility in $\mathbb{Q}[x]$
4. Prove that every proper ideal of a ring is contained in a maximal ideal.
5. Prove that every Euclidean domain is a principal ideal domain. Give an example of a unique factorization domain that is not a PID.
6. If R is Noetherian, and I is an ideal of R, then R/I is Noetherian.