

# Algebra Qual Prep: Summer, 2007.

## Linear Algebra

August 3, 2007

1. Finite Dimensional Vector Spaces
  - (a) Linear independence
  - (b) Span
  - (c) Basis/dimension
  - (d) Change of basis
2. Linear Transformations
  - (a) Invertibility
  - (b) Determinant
  - (c) Rank and nullity
  - (d) Characteristic polynomial
  - (e) Cayley-Hamilton Theorem
  - (f) Eigenvalues and eigenvectors
  - (g) Algebraic and geometric multiplicity
3. Canonical Forms and Decompositions
  - (a) Diagonal, unitary, real orthogonal, hermitian, normal matrices
  - (b) Similarity and diagonalization
  - (c) Gram-Schmidt orthonormalization
  - (d) Schur Decomposition
  - (e) QR decomposition
  - (f) Jordan canonical form
  - (g) Singular Value Decomposition
  - (h) Polar Decomposition
4. Variation of eigenvalues

- (a) Courant-Fischer Theorem (Weyl's Theorem)
- (b) Raleigh-Ritz Theorem
- (c) Cauchy Interlacing Theorem

5. Vector and matrix norms

- (a) Inner products
- (b) Matrix norms
- (c) Cauchy-Schwarz inequality
- (d) Triangle inequality
- (e) Equivalence
- (f) Completeness
- (g) Euclidean norm,  $l_p$ , max norm