

Algebra Qual Prep: Summer, 2008.

Practice Qual One

August 13, 2008

1. Suppose R is a ring (with 1) that contains a maximal ideal M such that every ideal of R contained in M is finitely generated. Show that every ideal of R is finitely generated.
2. Show \mathbb{Q} is not a finitely generated \mathbb{Z} -module.
3. If G is a finite group then define the *exponent* of G to be the least common multiple of the orders of the elements in G . Prove that the exponent of G is the order of G if and only if every Sylow subgroup of G is cyclic.
4. Suppose L is a splitting field of some polynomial over F and $p(x) \in F[x]$ is an irreducible polynomial. Show that if $p(x)$ has one root in L , then all other roots also lie in L .
5. If R is a commutative Noetherian ring and $\mathcal{N}(R)$ is the ideal of nilpotent elements of R , show that $\mathcal{N}(R)^t = \{0\}$ for some t .
6. If R is the ring of all rational numbers with odd denominators then prove that the intersection of all maximal ideals in R consists of all rational numbers with odd denominator and even numerator.