

Homework #2

- Simplify the expressions for $n(n-1)+(n-1)(n-2)+\dots+2\cdot 1$ and $1+2+\dots+(n-1)$. Then show the total number of additions/subtractions for Gaussian elimination with back substitution is $\frac{n^3}{3} + \frac{n^2}{2} - \frac{5n}{6}$.
 - Simplify the expressions for $[(n-1)+n(n-1)]+[(n-2)+(n-1)(n-2)]+\dots+[1+2\cdot 1]$ and $1+2+\dots+n$. Then show the total number of multiplications/divisions for Gaussian elimination with back substitution is $\frac{n^3}{3} + n^2 - \frac{n}{3}$.
- If one addition/subtraction is conducted in 0.001 seconds and one multiplication/division is conducted in 0.002 seconds, how many minutes are required to solve a linear system of 100 equations and 100 unknowns using Gaussian elimination with back substitution? How many days are required to solve a linear system of 1000 equations, 1000 unknowns?
- Solve the following linear system of equations using Gaussian elimination with back substitution,

$$\begin{aligned}2x_1 + 3x_2 + x_3 &= 0 \\x_1 + x_2 &= -1 \\4x_1 + 5x_2 - 2x_3 &= 2.\end{aligned}$$

Use augmented matrix form and label the elementary row operations performed.

- Solve the following linear system of equations using Gaussian elimination with back substitution,

$$\begin{aligned}2x_3 &= 4 \\x_1 + x_2 - x_3 &= -1 \\2x_1 - x_2 + x_3 &= -2.\end{aligned}$$

Use augmented matrix form and label the elementary row operations performed.

- For what value a will Gaussian elimination break down for the linear system of equations

$$\begin{aligned}2x_1 - 6ax_2 &= 3 \\3ax_1 - x_2 &= \frac{3}{2}?\end{aligned}$$

- Let A be a 4×4 matrix. Write down the matrix B such that

- (a) the third row of BA is 2 times the second row of A .
- (b) the second row of BA is the second row of A minus 3 times the fourth row of A .
- (c) the first row of BA is the fourth row of A and the fourth row of BA is the first row of A .

7. Solve the linear system of equations

$$\begin{aligned} 0.002x_1 + 59.14x_2 &= 59.16 \\ 5.291x_1 - 6.13x_2 &= 46.78, \end{aligned}$$

using 4 digit chopping and Gaussian elimination with back substitution. Find the relative errors for your calculations of x_1 and x_2 (exact solution is $x_1 = 10, x_2 = 1$).

8. Solve the linear system of equations

$$\begin{aligned} 0.002x_1 + 59.14x_2 &= 59.16 \\ 5.291x_1 - 6.13x_2 &= 46.78, \end{aligned}$$

using 4 digit chopping and Gaussian elimination with back substitution and partial pivoting. Find the relative errors for your calculations of x_1 and x_2 (exact solution is $x_1 = 10, x_2 = 1$).

9. What row interchange is performed in the first elimination step of Gaussian elimination on the linear system of equations

$$\begin{aligned} 2x_1 + 3x_2 + x_3 &= 0 \\ x_1 + x_2 &= -1 \\ 4x_1 + 5x_2 - 2x_3 &= 2. \end{aligned}$$

- (a) in partial pivoting. Why? What is the augmented matrix after pivoting?
- (b) in scaled partial pivoting. Why? What is the augmented matrix after pivoting?
- (c) For complete pivoting, what row and column interchange is performed? Why? What is the augmented matrix after pivoting?