

## Homework #1

- Write the following equations into two different fixed point problems.
  - $x^2 = 2$ .
  - $\sin x - x \cos x = 0$ .
- Plot  $y = g(x)$  and  $y = x$  to determine how many fixed points the following functions have.
  - $g(x) = x^3$ .
  - $g(x) = 1/x$ .
  - $g(x) = \sin x$ .
- Determine if the following  $g(x)$  satisfy  $g(x) \in [1, 2]$  for  $x \in [1, 2]$  by finding their maximum and minimum values.
  - $g(x) = -x^2 + x + 2$ .
  - $g(x) = 2/x$ .
  - $g(x) = x^3/2$ .
- Compute  $p_5, p_{10}, p_{15}$  of the fixed point iterations using the following  $g(x)$  with the following initial guess  $p_0$  (you may use a calculator without showing work).
  - $g(x) = \sqrt{x}$  with  $p_0 = 1/2$ .
  - $g(x) = \sqrt{x}$  with  $p_0 = 2$ .
  - $g(x) = \cos x$  with  $p_0 = 1/2$  radians.
- Let  $g(x) = 1/x + x/2$  and consider the interval  $[1.4, 1.45]$ .
  - Show  $g(x) \in [1.4, 1.5]$  for  $x \in [1.4, 1.45]$ .
  - Find  $0 < k < 1$  such that  $|g'(x)| \leq k$  for all  $x \in [1.4, 1.45]$ .
  - Use the  $k$  you just found to estimate the  $n$  such that  $p_n$  of fixed point iterations will have absolute error  $\leq 10^{-10}$  when  $p_0 = 1.425$ .
  - Compute  $p_3$  of the fixed point iteration using  $p_0 = 1.425$  and find the absolute error of this approximation (exact solution is  $\sqrt{2}$ ).
- Show if  $|g'(p)| < 1$ , then fixed point iterations will converge if  $p_0$  is close enough to  $p$  (use continuity of  $g'$  to show there is an interval  $[p - \epsilon, p + \epsilon]$  where  $|g'(x)| \leq k < 1$  and then try to use the theorem on convergence of fixed point iterations).
- Show if  $|g'(p)| > 1$ , then fixed point iterations will not converge (use continuity to show there is an interval  $[p - \epsilon, p + \epsilon]$  where  $|g'(x)| \geq k > 1$  and then consider the absolute error  $|p_n - p|$ ).

8. Create the file g.m:

```
function [gval] = g(x)
    gval = 1/x+x/2;
```

Also create the file fixedptit.m:

```
function [fixedpt] = fixedptit(x0,tol,N)
    x(1) = x0;
    n = 1;
    notstop = 1;
    while (notstop)
        n
        x(n+1) = g(x(n));
        if ((abs(x(n+1)-x(n)) < tol) | (n == N)) notstop = 0;
        else n = n+1;
    end
    end
    fixedpt = x(n);
```

Then run the command in Matlab:

```
>> fixedptit(1.425,0,3)
>> fixedptit(1.425,10-10,-1)
```

Write down your answers for both of these computations. Also write down the number of iterations (the  $n$ ) needed in the second computation.