

Homework #2

1. Consider approximating $2\sqrt{2}$ by finding the root of $f(x) = x^2 - 8$.
 - (a) Use Newton's method to generate p_3 starting with $p_0 = 3$.
 - (b) Write down the absolute error of this final approximation.
 - (c) Use the Secant method to generate p_4 starting with $p_0 = 2$ and $p_1 = 3$.
 - (d) Write down the absolute error of this final approximation.
2. Consider approximating $2\sqrt{2}$ by finding the root of $f(x) = x^2 - 8$.
 - (a) Use the method of false position to generate three approximations starting with the interval $[2, 3]$.
 - (b) Bound the error of this final approximation.
 - (c) Do you think you can bound the error of the 100th approximation without actually computing it (as you can in bisection method)?
3. Consider the problem of finding the point on the graph of $y = x^3$ closest to the point $(3, -1)$.
 - (a) Write down the expression for $d(x) =$ the square of the distance from (x, x^3) to $(3, -1)$.
 - (b) Minimize $d(x)$ by finding critical points: approximating the solution of $f(x) = d'(x) = 0$ using Newton's method to generate p_2 when $p_0 = 2$.
4. Let $f(x) = ax + b$ for some constants $a \neq 0$ and b .
 - (a) Show p_1 of Newton's method is the exact root for any chosen p_0 .
 - (b) Show p_2 of the Secant method is the exact root for any chosen p_0, p_1 .
5. Write down the formula for the Secant method with initial guesses $p_0 = a$ and $p_1 = b$ and simplify to show it is the same as the formula for the Secant method with initial guesses $p_0 = b$ and $p_1 = a$.
6. Let
$$f(x) = \begin{cases} \sqrt{x}, & \text{if } x \geq 0 \\ -\sqrt{-x}, & \text{if } x < 0. \end{cases}$$
 - (a) Generate Newton's method's p_1, p_2, p_3 in terms of $p_0 > 0$.
 - (b) Will Newton's method converge for any $p_0 \neq 0$? Why does this not violate the theorem on the convergence of Newton's method if the initial guess is close enough to the root?
7. Write down and simplify Newton's method's fixed point function $g(x)$ for the following root finding problems:

(a) $x^2 - 2 = 0$.

(b) $x^3 + x - 1 = 0$.

(c) $e^x \sin x = 0$.

8. Consider $f(x) = x^2 - 1$. Prove Newton's iteration converges for any $p_0 \in [3/4, 5/4]$.