

Homework #4

1. Generate the three more terms of Aitken's method on the following sequences:
 - (a) $p_n = 2^{-n}/n$ for $n = 1, 2, 3, \dots$
 - (b) $p_n = -1/n^2$ for $n = 1, 2, 3, \dots$
 - (c) $p_0 = 1$ radian and $p_n = \cos(p_{n-1})$ for $n = 1, 2, 3, \dots$
2. Generate the $p_0^{(1)}$ and $p_0^{(2)}$ elements of Steffensen's method using the following fixed point functions and initial guesses:
 - (a) $g(x) = \sqrt{x}$ and $p_0^{(0)} = 1/2$.
 - (b) $g(x) = \cos x$ and $p_0^{(0)} = 1$ radian.
 - (c) $g(x) = 1 + (\sin x)^2$ and $p_0^{(0)} = 2$ radians.
3. Given three initial guesses $p_0 = 1.4$, $p_1 = 1.3$, $p_2 = 1.1$ for Müller's method with $f(p_0) = 1.52$, $f(p_1) = 1.08$, $f(p_2) = 0.32$:
 - (a) Show the quadratic polynomial $p(x) = 2x^2 - x - 1$ passes through the points $(p_0, f(p_0))$, $(p_1, f(p_1))$, $(p_2, f(p_2))$.
 - (b) Use the quadratic formula to find the roots of $p(x)$.
 - (c) Which of these roots is chosen for the next approximation p_3 and why?
4. Suppose we know that $f(x) = x^5 + 3x^4 - 8x^3 - 12x^2 + 16x$ has integer roots. Repeatedly use the bisection method with either $a = -6, b = 3$ or $a = -6, b = 1.5$ and the method of deflation to get all 5 roots of $f(x)$. In the bisection method, you may stop when only one integer is in the bracketed interval and take that integer as the root for use in deflation.
5. For $n \geq 1$, suppose $P(x), Q(x)$ are polynomials of degree $\leq n$ and suppose $P(x) = Q(x)$ at $n + 1$ distinct points $x_i, i = 0, 1, \dots, n$. Prove $P(x) = Q(x)$ for all x by using the Fundamental Theorem of Algebra (Hint: Consider the polynomial $R(x) = P(x) - Q(x)$).
6. Write down and graph the linear Lagrange interpolating polynomial passing through data points $x_0 = -1, x_1 = 2$ when the underlying function is
 - (a) $f(x) = x^2$.
 - (b) $f(x) = e^x$.
7. Write down and graph the quadratic Lagrange interpolating polynomial passing through data points $x_0 = -1, x_1 = 0, x_2 = 2$ when the underlying function is
 - (a) $f(x) = x^2$.

(b) $f(x) = e^x$.

8. Create the file g.m:

```
function [gval] = g(x)
    gval = sqrt(x);
```

Also create the file steffensen.m:

```
function [approx] = steffensen(p0,N)
    p(1) = p0;
    for n = 1:N,
        p1 = g(p(n));
        p2 = g(p1);
        p(n+1) = p(n)-(p1-p(n))^2/(p(n)-2*p1+p2);
    end
    approx = p(N+1);
```

Then run the command in Matlab:

```
>> format long
>> steffensen(4,4)
```

and write down the output of the approximation and the quantity $|p_0^{(n+1)} - p|/|p_0^{(n)} - p|^2$ at each step. Then run your fixedptit.m program of HW #1 to find how many iterations will give comparable results.