

Homework #5

1. Use calculus to find the constant C such that

$$|f(x) - P(x)| \leq C$$

for all $x \in [0, 0.3]$, where $P(x)$ is the Lagrange interpolating polynomial for data with nodes $x_0 = 0, x_1 = 0.1, x_2 = 0.3$ and values from the underlying function $f(x) = e^{x+1}$.

2. Use calculus to find the constant C such that

$$|f(x) - P(x)| \leq C$$

for all $x \in [0, 1]$, where $P(x)$ is the piecewise linear interpolating polynomial for data with nodes $x_j = j/10, j = 0, \dots, 10$ and values from the underlying function $f(x) = x^2 + 1$.

3. Find n such that the piecewise linear interpolating polynomial for data with nodes $x_j = 2j/n, j = 0, \dots, n$ and values from the underlying function $f(x) = \sin x$ has absolute error in $[0, 2]$ less than 10^{-5} .
4. Suppose $x_0 = 0, x_1 = 0.4, x_2 = 0.7$ and $f[x_2] = 6, f[x_1, x_2] = 10, f[x_0, x_1, x_2] = 50/7$. Find the values of $f[x_0], f[x_1], f[x_0, x_1]$.
5. Consider the data points $(-2, 1), (-1, 4), (0, 11), (1, 16), (2, 13), (3, -4)$.
 - (a) Write down the divided difference table associated to this data.
 - (b) Determine from the table the degree of the Lagrange interpolating polynomial passing through these data points.
6. Write down the the Newton form of the Lagrange interpolating polynomial using $x_1 = 1, x_2 = 3, x_3 = 4, x_4 = 6$ and $f[x_3] = 5, f[x_3, x_2] = -1, f[x_3, x_2, x_4] = -2, f[x_3, x_2, x_4, x_1] = 3$.
7.
 - (a) Verify that $f[x_0, x_1, x_2] = f[x_2, x_0, x_1]$.
 - (b) Also verify that the Newton forms for the Lagrange interpolating polynomial using the data points with nodes $x_0 = a, x_1 = b$ and the data points with nodes in reverse order $x_0 = b, x_1 = a$ simplify to the same polynomial.
8. Consider the data points $(-2, -1), (0, 1), (-1, 3)$.
 - (a) Write down the Newton interpolatory divided difference form for this data.
 - (b) Add the data point $(1, -1)$ and write down the new Newton interpolatory divided difference form.
 - (c) Evaluate the polynomial of part (b) and its derivative at $x = 1/2$.

9. Suppose x_0, x_1, \dots, x_n are distinct numbers in the interval $[a, b]$ and f is $n + 1$ times differentiable in $[a, b]$. Prove for each $x \in [a, b]$, a number $\xi(x) \in (a, b)$ exists with

$$f(x) = P(x) + \frac{f^{(n+1)}(\xi(x))}{(n+1)!} (x - x_0)(x - x_1) \dots (x - x_n)$$

(Hint: See the proof in the book).

10. Enter Matlab and type:

```
>> help polyfit
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```
>> help polyval
```

Read these instructions on how the two functions work. Then enter:

```
>> x = 0:.1:1;
```

```
>> y = [0.95 0.23 0.6 0.48 0.89 0.76 0.45 0.01 0.82 0.44 0.61];
```

These are the data points. Then enter:

```
>> a = polyfit(x,y,10);
```

These are the coefficients of the Lagrange interpolating polynomial. Then enter:

```
>> plot(x,y,'o')
```

```
>> hold on
```

```
>> xfine = 0:.005:1;
```

```
>> plot(xfine,polyval(a,xfine))
```

```
>> axis([0 1 -3 3])
```

```
>> hold off
```

This is the Lagrange interpolating polynomial. Note the oscillations and the fact that this probably would not be the interpolating function you would draw through the points. Print this plot out and turn it in.