

Homework #5

- Given $n + 1$ data points with distinct nodes, prove there are an infinite number of different polynomials of degree $> n$ interpolating these data points (Hint: add a data point).
- Use nodes at the zeros of the degree three Chebyshev polynomial to construct an interpolating polynomial of degree ≤ 2 for the following functions in the interval $(-1, 1)$. Also, bound the maximum error of the approximation in $(-1, 1)$:
 - $f(x) = e^x$.
 - $f(x) = \sin x$.
- Use nodes at the zeros of the degree four Chebyshev polynomial to construct an interpolating polynomial of degree ≤ 3 for the following functions in the interval $(-1, 1)$. Also, bound the maximum error of the approximation in $(-1, 1)$:
 - $f(x) = e^x$.
 - $f(x) = \sin x$.
- Suppose $x_0 = 0, x_1 = 0.4, x_2 = 0.7$ and $f[x_2] = 6, f[x_1, x_2] = 10, f[x_0, x_1, x_2] = 50/7$. Find the values of $f[x_0], f[x_1], f[x_0, x_1]$.
- Consider the data points $(-2, 1), (-1, 4), (0, 11), (1, 16), (2, 13), (3, -4)$.
 - Write down the divided difference table associated to this data.
 - Determine from the table the degree of the interpolating polynomial of least degree passing through these data points.
- Simplify $f[x_0, x_1, x_2]$ and $f[x_2, x_0, x_1]$ and verify they are the same.
 - Also verify that the Newton forms for the interpolating polynomials of least degree using the data points with nodes $x_0 = a, x_1 = b$ and the data points with nodes in reverse order $x_0 = b, x_1 = a$ simplify to the same polynomial.
- Consider the data points $(-2, -1), (0, 1), (-1, 3)$.
 - Write down the Newton form for the interpolating polynomial for this data.
 - Add the data point $(1, -1)$ and write down the new Newton form.
 - Evaluate the polynomial of part (b) and its derivative at $x = 1/2$.
- (Matlab) Write a program that inputs m ; two vectors of data points x and y , both with m components; and a location z . Have this program output the value of the Newton form of the interpolating polynomial at the location z .