Homework #6

1. Show that for any z,
$$f[x_0, x_1, \ldots, x_n, z] = \frac{f^{(n+1)}(\xi(z))}{(n+1)!}$$
by consider the error formula of the Lagrange interpolatory polynomial passing through the $n+1$ data points at $x_0, x_1, \ldots, x_n$ and the Newton interpolatory divided difference form of the Lagrange interpolatory polynomial with the additional data point at z, both evaluated at z.

2. Consider the Hermite interpolating polynomial
$$H(x) = \sum_{j=0}^{n} f(x_j)H_{n,j}(x) + \sum_{j=0}^{n} f'(x_j)\hat{H}_{n,j}(x).$$
Use the forms of $H_{n,j}$ and $\hat{H}_{n,j}$ to show:
   (a) $H_{n,j}(x_k) = 0$ for all $k \neq j$ and $H_{n,j}(x_j) = 1$.
   (b) $\hat{H}_{n,j}(x_k) = 0$ for all k.
   (c) $H'_{n,j}(x_k) = 0$ for all k.
   (d) $\hat{H}'_{n,j}(x_k) = 0$ for all $k \neq j$ and $\hat{H}'_{n,j}(x_j) = 1$.

3. Consider $x_0 = -1, x_1 = 1$ and $f(x_0) = 0, f(x_1) = 2$ and $f'(x_0) = 1, f'(x_1) = -1$.
   (a) Find the Hermite interpolating polynomial using divided differences.
   (b) Add the information $x_2 = 0$ and $f(x_2) = 1$ and $f'(x_2) = 0$ and find the resulting Hermite interpolating polynomial.
   (c) Approximate $f(1/2)$ and $f'(1/2)$ using the results of part (b).

4. Consider $x_0 = 0, x_1 = 1, x_2 = 2$ and $f(x_0) = 0, f(x_1) = 1, f(x_2) = 2$ and $f'(x_0) = 0, f'(x_1) = 1, f'(x_2) = 2$. Find the Hermite interpolating polynomial that interpolates this data and use it to approximate $f(3/2)$.

5. (a) Verify that $H(x) = x^3 + 3x^2 - x - 2$ is the Hermite interpolating polynomial for the data $x_0 = 0,\ f(x_0) = -2,\ f'(x_0) = -1$ and $x_1 = 1,\ f(x_1) = 1,\ f'(x_1) = 8$.
   (b) Add the data $x_2 = 2,\ f(x_2) = 0,\ f'(x_2) = 0$ and find the resulting Hermite interpolating polynomial.

6. Consider the underlying function $f(x) = \sin x$ and nodes $x_0, x_1, \ldots, x_n \in [0, 2\pi]$ satisfying $x_j = 2\pi j/n$.
   (a) In the case $n = 10$, use error bounds to bound the absolute error between $f(1)$ and $H(1)$, where $H(x)$ is the piecewise cubic Hermite interpolating polynomial.
   (b) Using error bounds, determine n such that the absolute error between $f(x)$ and the piecewise cubic Hermite interpolating polynomial $H(x)$ will be less than 10^{-10}.