Homework #8

1. (a) Find $(x(t), y(t))$ with $0 \leq t \leq 1$ for the Bezier curve interpolating points $(0,0), (1,0)$ with respective guidepoints $(0,2), (1,2)$.

   (b) Use the formula on page 162 to do the same when the guidepoints are now $(0,2), (1,-2)$.

2. (a) Consider the data

$$(-1, 0.03), (-1, -0.05), (0, 1.03), (0.5, 1.48), (1, 1.96)$$

   Write down the normal equations for linear least squares.

   (b) Solve the normal equations to get the best fitting line in the least squares sense.

3. Consider the data

$$(0, y_1), (0, y_2), \ldots, (0, y_n).$$

   Find the constant function C best fitting this data in the least squares sense. What is another name for this constant?

4. Consider the data

$$(0, y_1), (0, y_2), \ldots, (0, y_n)$$

   with $y_1 < y_2 < \ldots < y_n$ and with n odd. Find the constant function C best fitting this data in the sense of minimizing

$$E(C) = \sum_{i=1}^{n} |y_i - C|.$$

   What is another name for this constant? (Hint: Look at E separately in each interval $[y_i, y_{i+1}]$ and determine from its derivative whether it is increasing or decreasing. Then try to piece together this information to conclude something about the minimum of E)

5. (a) Consider the data

$$(-1, 2.04), (-0.5, 1.23), (0, 1.01), (0.5, 1.28), (1, 1.99)$$

   Write down the normal equations for quadratic least squares.

   (b) Solve the normal equations to get the best fitting parabola in the least squares sense.

6. Enter Matlab and type:

   >> x = 0:.1:1;

   >> y1 = [0.95 0.23 0.6 0.48 0.89 0.76 0.45 0.01 0.82 0.44 0.61];

>> y2 = [-2 0.23 0.6 0.48 0.89 0.76 0.45 0.01 0.82 0.44 0.61];

These are two sets of data points, with one differing from the other just at its initial value. Then enter:

```
>> cs1 = spline(x,[0 y1 0]);
>> cs2 = spline(x,[0 y2 0]);
>> xfine = 0.5:.005:1;
>> plot(xfine,abs(ppval(cs1,xfine)-ppval(cs2,xfine)))
```

The plot shows the error in $[0.5, 1]$ of the clamped cubic splines with zero first derivative at the endpoints. Print this plot out and turn it in.

7. Plot the Bezier curve for points $(0, 0), (1, 0)$ and respective guidepoints $(0, 1), (1, 1)$ through:

```
>> t = 0:.005:1;
>> x = -2*t.^3+3*t.^2;
>> y = -3*t.^2+3*t;
>> subplot(221)
>> plot(x,y)
```

The continue to plot the Bezier curve for the same points with respective guidepoints $(0, 1), (1, -1)$ through:

```
>> x = -2*t.^3+3*t.^2;
>> y = 6*t.^3-9*t.^2+3*t;
>> subplot(222)
>> plot(x,y)
```

Then for subplot(223) and subplot(224), plot the Bezier curves of problem #1. Finally, print out the plot and turn it in.

8. Enter Matlab and type:

```
>> x = 0:.1:1;
>> y = [0.95 0.23 0.6 0.48 0.89 0.76 0.45 0.01 0.82 0.44 0.61];
```

These are the data points. Then type:

```
>> a = polyfit(x,y,1);
```

This gives the coefficients of the best fitting line in the least squares sense. Then enter:

```
>> subplot(121)
>> plot(x,y,'o')
>> hold on
>> xfine = 0:.005:1;
>> plot(xfine,polyval(a,xfine))
>> axis([0 1 -0.5 1.5])
>> hold off
```

Continue with:

```
>> a = polyfit(x,y,2);
```

This gives the coefficients of the best fitting parabola in the least squares sense. Then enter:

```
>> subplot(122)
>> plot(x,y,'o')
>> hold on
>> plot(xfine,polyval(a,xfine))
>> axis([0 1 -0.5 1.5])
>> hold off
```

Print this plot out and turn it in.