

Homework #1

1. Suppose $\vec{x} = (x_1, x_2, \dots, x_n)$ is given. Consider the following algorithm

Step 1. Set $y = 0$.

Step 2. For $i = 1$ to n , do step 3

Step 3. Set $y = y + x_i$.

Step 4. Set $y = y/n$.

Step 5. Output y .

- What does this algorithm do?
- Count the number of additions/subtractions this algorithm performs. When $n \rightarrow \infty$, this number is $\mathcal{O}(n^p)$ for what p ?
- Count the number of multiplications/divisions this algorithm performs. When $n \rightarrow \infty$, this number is $\mathcal{O}(n^p)$ for what p ?

2. Suppose the vector $n \times 1$ vector \vec{x} and the $n \times n$ matrix $A = (a_{ij})$ are given. Consider the following algorithm:

Step 1. For $i = 1$ to n , do steps 2 to 5.

Step 2. Set $y_i = 0$

Step 3. For $j = 1$ to n , do step 4.

Step 4. Set $y_i = y_i + a_{ij}x_j$.

Step 5. Output y_i .

- What does this algorithm do?
- Count the number of additions/subtractions this algorithm performs. When $n \rightarrow \infty$, this number is $\mathcal{O}(n^p)$ for what p ?
- Count the number of multiplications/divisions this algorithm performs. When $n \rightarrow \infty$, this number is $\mathcal{O}(n^p)$ for what p ?

3. Find the absolute and relative errors in the following.

- exact is 0.54112, approximation is 0.54110.
- exact is 400, approximation is 402.
- exact is -3.4212 , approximation is -3 .

4. Suppose given any real number x , we drop everything after the decimal point to arrive at an integer n . Think of n as an approximation of x .

- Bound the absolute error (find C such that the absolute error $\leq C$).
- Bound the relative error (find C such that the relative error $\leq C$).