

Homework #8

1. Consider $u_t = ((\sin x + 2)u_x)_x$ with $x \in [-\pi, \pi]$ and Dirichlet boundary conditions $u(-\pi, t) = u(\pi, t) = 0$ and initial condition $u(x, 0) = \sin x$. Use

$$\frac{U_j^{n+1} - U_j^n}{\Delta t} = \frac{C_{j+1/2} \frac{U_{j+1}^n - U_j^n}{\Delta x} - C_{j-1/2} \frac{U_j^n - U_{j-1}^n}{\Delta x}}{\Delta x}$$

with $C(x) = \sin x + 2$ and $\Delta x = 2\pi/100$ and $\Delta t = \Delta x^2 / (2 \sup_{x \in [-\pi, \pi]} C(x))$ in Matlab to plot the solution after 500 time steps.

2. Consider $u_t + (x^2 - 1)u_x = 0$ with $x \in [-\pi, \pi]$ and no boundary conditions and initial condition $u(x, 0) = \sin x$. Use

$$\frac{U_j^{n+1} - U_j^n}{\Delta t} + \max\{C_j, 0\} \frac{U_j^n - U_{j-1}^n}{\Delta x} + \min\{C_j, 0\} \frac{U_{j+1}^n - U_j^n}{\Delta x} = 0$$

with $C(x) = x^2 - 1$ and $\Delta x = 2\pi/100$ and $\Delta t = \Delta x / \sup_{x \in [-\pi, \pi]} |C(x)|$ in Matlab to plot the solution after 10, 30, and 50 time steps.

3. Consider $u_t = u_{xx}$ with $x \in [-\pi, \pi]$ and Dirichlet boundary conditions $u(-\pi, t) = u(\pi, t) = 0$ and initial condition $u(x, 0) = \sin x$. Use Crank-Nicolson

$$\frac{U_j^{n+1} - U_j^n}{\Delta t} = \frac{1}{2} \left[\frac{U_{j+1}^n - 2U_j^n + U_{j-1}^n}{\Delta x^2} + \frac{U_{j+1}^{n+1} - 2U_j^{n+1} + U_{j-1}^{n+1}}{\Delta x^2} \right]$$

with $\Delta x = 2\pi/100$ and $\Delta t = \Delta x$ in Matlab to plot the solution after 10, 30, and 50 time steps.

4. (a) The approximation generated in problem #1 is approximating the exact solution at what time t ?
- (b) Count the exact number of multiplications performed in problem #1. Then divide this number by the time in part (a) to get the number of multiplications per 1 unit of time.
5. (a) The approximation generated in problem #2 is approximating the exact solution at what time t ?
- (b) Count the exact number of multiplications performed in problem #1. Then divide this number by the time in part (a) to get the number of multiplications per 1 unit of time.
6. (a) The approximation generated in problem #3 is approximating the exact solution at what time t ?
- (b) Count the exact number of multiplications performed in problem #1. Then divide this number by the time in part (a) to get the number of multiplications per 1 unit of time.

7. Consider

$$\frac{U_{jk}^{n+1} - U_{jk}^n}{\Delta t} = C \left[\frac{U_{j+1,k}^n - 2U_{jk}^n + U_{j-1,k}^n}{\Delta x^2} + \frac{U_{j,k+1}^n - 2U_{jk}^n + U_{j,k-1}^n}{\Delta y^2} \right].$$

Let $\Delta x = \Delta y$. Use von Neumann analysis, plugging in $U_{jk}^n = g^n e^{ij\theta} e^{ik\psi}$ to find a C_0 such that $\Delta t = C_0 \Delta x^2$ gives stability.