Homework #9

1. Consider $u_t = u_{xx} + u_{yy}$ for $(x, y) \in (0, 1) \times (0, 1)$ with $u(x, y, 0) = 0$ for $(x, y) \in (0, 1) \times (0, 1)$ and $u(0, y, t) = 0$, $u(1, y, t) = 100$, $u(x, 0, t) = u(x, 1, t) = 100x$. Use first order forward differencing in time and second order central differencing in space with $\Delta x = \Delta y = 0.01$ and $\Delta t = \Delta x^2/4$ in Matlab to plot the solution after 100 and 500 time steps. You can use the "mesh" command to plot.

2. Consider $u_t + u_x + u_y = 0$ for $(x, y) \in (0, 1) \times (0, 1)$ with $u(x, y, 0) = 0$ for $(x, y) \in (0, 1) \times (0, 1)$ and $u(0, y, t) = 1$, $u(x, 0, t) = 1$. Use first order forward differencing in time and first order backward differencing in space with $\Delta x = \Delta y = 0.01$ and $\Delta t = \Delta x/2$ in Matlab to plot the solution after 20 and 100 time steps. You can use the "mesh" command to plot.

3. Consider $u''(x) = e^x$ for $x \in (0, 1)$ with $u(0) = 0$ and $u(1) = 0$. Write down the linear system involved in solving this PDE when second order central differencing is used with $\Delta x = 0.2$.

4. Consider $u''(x) = 0$ for $x \in (0, 1)$ with $u(0) = 0$ and $u(1) = 100$. Use second order central differencing with $\Delta x = 0.01$ in Matlab to plot the solution.

5. Consider $u''(x) + Cu(x) = f(x)$ for $x \in (a, b)$ with $u(a) = 0$ and $u(b) = 0$ and C constant. Find the Ritz and Galerkin formulations of this problem.

6. Consider $u''(x) = e^x$ for $x \in (0, 1)$ with $u(0) = 0$ and $u(1) = 0$. Take the grid with nodes at $0, 0.2, 0.5, 0.6, 0.7, 1$ but numbered as $x_0 = 0.7, x_1 = 0.2, x_2 = 0.5, x_3 = 0.6$. Write down the linear system to solve in the finite element method with piecewise linear elements in the form
$$A\vec{\alpha} = \vec{b},$$
where $\vec{\alpha} = [\alpha_0, \alpha_1, \alpha_2, \alpha_3]^t$ and the approximate solution equals
$$\alpha_0\phi_0(x) + \alpha_1\phi_1(x) + \alpha_2\phi_2(x) + \alpha_3\phi_3(x)$$
with $\phi_j(x)$ the basis element corresponding to the node x_j.

7. Consider $u''(x) = 0$ for $x \in (0, 1)$ with $u(0) = 0$ and $u(1) = 100$. Use the finite element method with piecewise linear elements over the grid with stepsize $h = 0.01$ used in $[0, 1/2]$ and stepsize $h = 0.02$ used in $[1/2, 1]$.