Math 172 Midterm

May 3, 2006

- Please put your name, ID number, and sign and date.
- There are 4 problems worth a total of 100 points.
- **You must show your work to receive credit**.

Print Name: _____

Student ID: _____

Signature and Date: _____

Problem	Score
1	/25
2	/25
3	/25
4	/25
Total	/100

1. (25 pts) Write down a consistent approximation scheme for the partial differential equation

$$u_t(x,t) = Bu_{xx}(x,t) + Au_x(x,t) + Cu(x,t) + f(x,t).$$

(It does not have to be convergent and you do not have to prove consistency)

2. (25 pts) Suppose $|u''(x)| \leq M$ for all x. Prove

$$\left| \frac{u(x+h) - u(x)}{h} - u'(x) \right| \leq Const \cdot h.$$

3. (25 pts) Use energy estimates to prove the problem

$$\begin{cases} u_t(x,t) = u_{xx}(x,t) + u_x(x,t), \quad x \in [a,b], t \in [0,T] \\ u(a,t) = 0 \\ u(b,t) = 0 \\ u(x,0) = u_0(x) \end{cases}$$

is well-posed.

4. (25 pts) Prove the formula

$$\frac{U_k^{n+1} - U_k^n}{\Delta t} + \frac{U_{k+1}^n - U_{k-1}^n}{2\Delta x} = 0$$

is never stable when $\Delta t/\Delta x = Const.$