Math 20A.
Final Examination
December 11, 2007

Turn off and put away your cell phone.
No calculators or any other electronic devices are allowed during this exam.
You may use one page of notes, but no books or other assistance during this exam.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

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1. For each of the following limits, either compute it or explain why it doesn’t exist.

(a) (3 points) \( \lim_{x \to 2} \frac{|x - 2|}{x - 2} \)

(b) (3 points) \( \lim_{x \to 0} \frac{2^x - 1}{x} \)

(c) (3 points) \( \lim_{x \to 0} \left[ \sin(x) \cos\left(\frac{1}{x}\right) \right] \) (Hint: apply the Squeeze Theorem.)
2. Let \( f(x) \) be the continuous function on \([-4, 4]\) whose graph is given below and let \( F(x) = \int_{-2}^{x} f(t) \, dt \).

\[
\begin{array}{c}
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\includegraphics[width=0.5\textwidth]{graph.png}
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(a) (3 points) Evaluate \( F(2) \).

(b) (2 points) Evaluate \( F'(-3) \).

(c) (3 points) Over what intervals shown on the graph is \( F \) concave down? Explain your answer.
3. Evaluate the following derivatives.

(a) (2 points) \[ \frac{d}{dx} \cos\left(\frac{x}{x + 2}\right) \]

(b) (2 points) \[ \frac{d}{dx} \left\{ \sqrt{\sin(4x) + \pi^2} \right\} \]

(c) (2 points) \[ \frac{d}{dx} x \sqrt{x + \sqrt{x}} \]

(d) (2 points) \[ \frac{d}{dx} \int_0^{x^2} e^{4t^2} \, dt \]
4. Let \( f(x) = 5 \ln(x^2 + 4) - 2x \); then, \( f'(x) = \frac{2(x^2 - 5x + 4)}{x^2 + 4} \).

(a) (2 points) Find the critical points of \( f(x) \).

(b) (3 points) Determine the interval(s) on which \( f(x) \) is increasing and the interval(s) on which \( f(x) \) is decreasing.

(c) (3 points) For each critical point found in part (a), indicate whether it is a local minimum, local maximum or neither without computing \( f''(x) \). Be sure to state how you arrived at your conclusion(s).
5. (6 points) Compute the area bounded by the graph \( y = x |x - 2| \), the \( x \)-axis, the \( y \)-axis and the vertical line \( x = 3 \).
6. Let \( f(x) = (3x - x^2) e^{2x} \).

(a) (3 points) Find the linear approximation to \( f(x) \) near \( x = 5 \).

(b) (3 points) Using the result of part (a), estimate by how much \( f(x) \) changes if \( x \) changes from 5 to 4.75.
7. Let \( f(x) = 2 + \frac{1}{x} \)

(a) (4 points) Using only the definition of a derivative (and without applying any differentiation rules), compute \( f'(3) \).

(b) (2 points) Find the equation of the tangent line to the graph of \( f(x) \) at the point \( x = 2 \).
8. (6 points) A farmer has $L$ feet of fencing with which to fence a rectangular corral with six compartments (see the figure below). What are the dimensions ($x$ and $y$) of the corral with maximum area that the farmer can fence, and what is the maximum area?
9. (6 points) The volume of a cube is increasing at a rate of 1 cm³/min. How fast is the surface area increasing when the length of an edge is 2 cm?