

Section _____ Print Name _____

Signature _____ ID # _____

- Use this exam for your work. **Do NOT use a blue book.**
- Use the back of the exam pages if you need scratch paper.
- The exam is CLOSED BOOK, but you may use ONE SHEET OF NOTES.
- Calculators are NOT allowed.
- **You must show your work to receive credit.**

1. (40 pts.) Let \mathcal{R} be the region between the two parabolas $y = x^2$ and $x = y^2$.

Let \mathcal{V} be the volume obtained when \mathcal{R} is rotated about the y -axis.

- (a) Sketch the region \mathcal{R} . Include in your drawing the coordinates of the point where the parabolas intersect.

Set up (but *do not evaluate*) integrals for the following.

(b) The arc length of the boundary of \mathcal{R} that is on the parabola $y = x^2$.

(c) The volume of \mathcal{V} .

(d) The surface area of \mathcal{V} . *Be careful:* \mathcal{V} has what might be called an inner and outer surface. The surface area is the sum of the areas of these two surfaces.

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2. (20 pts.) Given the two curves $r = 2$ and $r = 4 \cos \theta$ in polar coordinates.
- (a) Find the polar coordinates of the points where the curves intersect.
- (b) Set up (but *do not evaluate*) an integral for the area that lies inside the curve $r = 4 \cos \theta$ but outside the curve $r = 2$; that is, the area of the region for which $2 \leq r \leq 4 \cos \theta$.

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3. (30 pts.) Express each of the following in the form $a + bi$ with a and b real numbers.

Do **NOT** leave trig functions in your answers.

(a) $(2 + 4i)/(1 - 7i)$

(b) $(2\sqrt{3} + 2i)^{20}$

(c) $e^{3+i\pi/2}$

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4. (20 pts.) (a) Determine whether $\int_0^{\infty} e^{-x} dx$ is convergent or divergent.

(b) Use part (a) and the comparison theorem to determine whether $\int_0^{\infty} \frac{e^{-x}}{2 + \sin x} dx$ is convergent or divergent.

5. (30 pts.) Evaluate the following integrals.

(a) $\int \frac{\ln x}{x^2} dx$

(b) $\int \cos x \cos(3x) dx$

(c) $\int \frac{dx}{x^2 \sqrt{1-x^2}}$

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6. (10 pts.) Write out the partial fraction decomposition of the function $\frac{x}{x^2 - 1}$.

7. (15 pts.) Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + 1}{x^2 y}$ with the initial condition $y(1) = -2$ for y as a *function* of x ; that is, find $y(x)$.

8. (15 pts.) Use Euler's method with step size 1.0 to estimate $x(3.0)$, where $x(t)$ is the solution of the initial value problem

$$\frac{dx}{dt} = x + t, \quad x(0) = 0.$$

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9. (20 pts.) Consider the integral $I = \int_0^2 \frac{2}{4x+1} dx$.

(a) Use the Midpoint rule with $n = 2$ subintervals to approximate I .

(b) How large should n be so that the midpoint approximation of I is accurate to within 6×10^{-4} ?