Math 20d, Winter **Midterm Exam II (Version 1)** **May, 2007**
Instructor: Professor Ni

Name:	____________________
Student #:	____________________
TA's Name:	____________________
Session #:	____________________

INSTRUCTIONS

1. **Name of your TA worths 10 bonus pts.**
1. **NO CALCULATOR.**
2. **CLOSE BOOK, CLOSE NOTES.**
3. **ID WILL BE CHECKED. GET IT READY!**

Problem	Points
page 2 (15 points)	
Page 3 (15 points)	
Page 4 (15 points)	
Page 5 (15 points)	
Page 6 (15 points)	
Page 7 (15 points)	
Bonus (10 points)	
Total (100 points)	

Notations: $y' = dy/dt$ or dy/dx; $y'' = d^2y/dt^2$, or d^2y/dx^2.

Filling the blank question: We do want to see the key steps in case that your answer is correct. Correct answer without steps only earns partial credits. Incorrect answer gets 0 credit.

1. For each of the following determine a suitable form for $Y(t)$ if the method of undetermined coefficients is to be used.

   a) $y'' + 4y = t^2 \sin 2t$.

   Then $Y(t)$ should be the form __________________________.

   b) $y'' - 4y' + 4y = 2t^2 + 2te^{2t}$.

   Then $Y(t)$ should be of the form __________________________.

2. The position of a certain spring-mass system satisfies the initial value problem:

$$\frac{5}{2}y'' + ku = 0, \qquad u(0) = 2, \quad u'(0) = v.$$

If the period and amplitude of the resulting motion are observed to be π and 3, respectively, determine the value of k and v.

$k =$ ________________________

$v =$ ________________________

Regular problems: For the following problems we give partial credits for correct and relevant steps.

3. Two methods of reduction of order were introduced in the class. Use any of them to find general solution of the equation

$$(x-1)y'' - xy' + y = 0$$

with $y_1(x) = e^x$.

4. Consider vectors

$$\vec{X}^{(1)}(t) = \begin{pmatrix} t \\ 1 \end{pmatrix}, \qquad \vec{X}^{(2)}(t) = \begin{pmatrix} t^2 \\ 2t \end{pmatrix}.$$

a) Compute the Wronskian of them.

b) In what intervals are they linear independent.

5. a) Find the general solution of

$$\vec{X}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \vec{X}.$$

b) Is $\vec{0}$ a saddle point or a node?

6. a) Find the solution of

$$\vec{X}' = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix} \vec{X}, \quad \vec{X}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

b) Describe the behavior of the solution as $t \to \infty$.

END OF EXAM