**Math 20d, Fall** **Midterm Exam I (Green)** **2007**
**Instructor: Professor Ni**

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| Name: | Key |
| Student #: | Solution |
| TA's Name: | Solution |
| Session #: | |

**INSTRUCTIONS**

1. **NO CALCULATOR.**
2. **CLOSE BOOK, CLOSE NOTES.**
3. **ID WILL BE CHECKED. GET IT READY!**

| Problem | Points |
|---|---|
| page 2 (50 points) | |
| Page 3 (20 points) | |
| Page 4 (20 points) | |
| Page 5 (10 points) | |
| Total (100 points) | |

Notations: $y' = dy/dt$ or $dy/dx$; $y'' = d^2y/dt^2$, or $d^2y/dx^2$.

**Multiple choices questions:** Questions 1 -5 are multiple choices. Circle all correct answers. There may be more than one correct answer listed. Incorrect responses will be penalized accordingly. For example, if problem X has 2 correct answers and if someone chooses two correct ones and one incorrect (total circle three answers) then he will only receives 5 out of 10 since the incorrect one cancels the credit of one of the two correct choices.

1. Let $y_0$ be the initial value of the problem $y' - y = 1 + 3\sin t$, $y(0) = y_0$. For which choice(s) of $y_0$, the solution $y(t)$ stays bounded as $t \to \infty$.

   d

   (a) $y_0 = 0$.

   (b) $y_0 = \frac{5}{2}$.

   (c) $y_0 = -5$.

   (d) $y_0 = -\frac{5}{2}$.

   (e) None of them above.

2. Consider the initial value problem $y' = y^{1/3}$, $y(0) = 0$. Select the point(s) (on $ty$-plane through which there will be a solution to this initial value problem passing,

   c,d

   (a) $(1/2, 1)$.

   (b) $(1, 1)$.

   (c) $(2, 1)$.

   (d) $(3, 1)$.

   (e) None of them above.

3. From equations below select the homogeneous equation(s).

   b,c,d

   (a) $y' - y = 0$.

   (b) $xy' - y = 0$.

   (c) $y' = \frac{y-4x}{x-y}$.

   (d) $y' = \frac{y^2-4x^2}{y^2}$.

   (e) None of them above.

4. From below select the first order exact equation(s).

   a, c, d

   (a) $2x + y^2 + 2xyy' = 0$.

   (b) $3xy + y^2 + (x^2 + xy)y' = 0$.

   (c) $y\cos x + 2xe^y + (\sin x + x^2e^y - 1)y' = 0$.

   (d) $y' = -\frac{4x}{y}$.

   (e) None of the above.

5. Select the initial value problem(s) which is certain to have one unique solution (meaning that one and only one solution exists).

   a,d

   (a) $y' = -\frac{4t}{y}, \quad y(0) = 1$.

   (b) $y' = y^{1/3}, \quad y(0) = 0$.

   (c) $y' = t^2\tan(y), \quad y(1) = \frac{\pi}{2}$.

   (d) $(x-2)y'' + y' + (x-2)(\tan x)\, y = 0, \quad y(3) = 1, \quad y'(3) = 2$.

   (e) None of them above.

**Filling the blank question:** We do want to see the key steps in case that your answer is correct. Correct answer without steps only earns partial credits. Incorrect answer gets 0 credit.

6. Without attempting to solve the equation determine the maximum interval in which the solution is certain to exist.

   a) $(4-t^2)y' + 2ty = 3t^2, \quad y(-3) = 1.$

   The interval is (-oo, -2).

   b) $(x-3)y'' + xy' + (\ln|x|)y = 0, \quad y(1) = 0, \quad y'(1) = 1.$

   The interval is (0, 3).