
1. Adding the two equations together gives $3x - y + 2 = 0$ so $y = 3x + 2$. Inserting this in the first equation gives $z = 3y - 2x + 1 = 3(3x + 2) - 2x + 1 = 7x + 7$. Hence the equation of the plane is $(x, y, z) = (t, 3t + 2, 7t + 7) = (1, 3, 7)t + (0, 2, 7)$.

2. (a). $F(x, y, z) = \nabla \phi = y^2i + 2xyj$.
(b). Along the curve we have $\frac{dx}{y^2} = \frac{dy}{2xy} = \frac{dz}{0}$ so $ydy = 2xdx$ and $dz = 0$. Hence $y^2 = 2x^2 + C$ so $y = \pm\sqrt{2x^2 + C_1}$ and $z = C_2$ Since the curve starts at $(1, -1, 1)$ we must have $(x, y, z) = (t, -\sqrt{2t^2 - 1}, 1)$

3. (a). $\phi_x = ze^{x-y}$, $\phi_y = -ze^{x-y}$ and $\phi_z = e^{x-y}$. Integrating these equations give $\phi = ze^{x-y} + h_1(y, z)$, $\phi_y = ze^{x-y} + h_2(x, z)$ and $\phi = ze^{x-y} + h_3(x, y)$ which can all be satisfied if we pick $h_1 = h_2 = h_3 = 0$ in which case $\phi = ze^{x-y}$.
(b) The end point of the curve is $(1, 0, 0)$ and the initial point is $(0, 0, 1)$ so $\int_C F \cdot R = \phi(1, 0, 0) - \phi(0, 0, 1) = -1$.
(c) If $G$ denotes the vector field then $\nabla \times G = 0$ and $G$ is continuously differentiable in the region obtained by removing the origin from space. Since this region is simply connected it follows that there exists a potential so $G$ is conservative. Solving the equations we see that indeed $\psi = \ln \sqrt{x^2 + y^2 + z^2}$ is a potential so it is conservative.

4. (a) See handout.
(b) $R = \{(u, v); 1 \leq v^2 \leq 2, 2 \sinh u \leq \cosh u \leq 3 \sinh u\}$. The last two inequalities means that $e^u - e^{-u} = (e^u + e^{-u})/2 \leq 3(e^u - e^{-u})/2$ and multiplying by $e^u$ gives $e^{2u} - 1 \leq e^{2u}/2 + 1/2 \leq 3e^{2u}/2 - 3/2$ or $e^{2u}/2 \leq 3/2$ respectively $2 \leq e^{2u}$ so $2 \leq e^{2u} \leq 3$. or $\ln 2 \leq 2u \leq \ln 3$. We get $R = \{(u, v); \ln 2/2 \leq u \leq \ln 3/2, 1 \leq v \leq \sqrt{2}\}$.
(c) $\frac{\partial (x, y)}{\partial (u, v)} = -v$.
(d) $\int \int_R \frac{dxdy}{(1 + x^2 - y^2)^2} = \int \int_R \frac{vdu dv}{(1 + v^2)^2} = \int_0^{b-1/2} \frac{du}{1 + v^2} = \frac{a - b}{2} \left( \frac{1}{1 + d^2} - \frac{1}{1 + c^2} \right)$
5. (a) See book.  
   (b) \( A = \text{Area}(D) = \frac{1}{2} \int_C x \, dy - y \, dx \)
   
   (c) \( A = \frac{1}{2} \int_C x \, dy - y \, dx = \frac{1}{2} \int_{-\sqrt{3}}^{\sqrt{3}} t^2 (t^2 - 1) - \left( \frac{t^3}{3} - t \right) 3 \, dt = \int_{-\sqrt{3}}^{\sqrt{3}} t^2 \, dt \)
   \[ = \frac{t^3}{3} \Big|_{-\sqrt{3}}^{\sqrt{3}} = 2\sqrt{3} \]
   
   (d) \( L = \int_C ds = \int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{(dx/dt)^2 + (dy/dt)^2} \, dt = \int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{2t^2 + (t^2 - 1)^2} \, dt \)
   \[ = \int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{t^2 + 1} \, dt = \frac{t^3}{3} + t \Big|_{-\sqrt{3}}^{\sqrt{3}} = 4\sqrt{3} \]

6. (a) See book.  
   (b) The exterior unit normal is \( \mathbf{n} = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})/a. \)
   
   \[ \iint_S \mathbf{F} \cdot \mathbf{n} \, dS = \iint_S \frac{x + y + z^4}{a} \, dS = \int_0^\pi \int_0^{2\pi} \frac{a \sin \phi \cos \theta + a \sin \phi \sin \theta + a^4 \cos^4 \phi}{a} \cdot a^2 \sin \phi \, d\theta d\phi \]
   \[ = \int_0^\pi \int_0^{2\pi} 2\pi a^5 \cos^4 \phi \sin \phi \, d\phi = 2\pi a^5 \frac{\cos^5 \phi}{5} \bigg|_0^\pi = \frac{4\pi a^5}{5} \]
   
   (c) Let \( B \) be the ball of radius \( a. \)
   
   \[ \iint_S \mathbf{F} \cdot \mathbf{n} \, dS = \iiint_B \text{div} \mathbf{F} \, dV = \iiint_B 3z^2 \, dV = \int_0^\pi \int_0^{2\pi} \int_0^a 3r^2 \cos^2 \phi \rho^2 \sin \phi \, dr d\theta d\phi \]
   \[ = \int_0^\pi \int_0^{2\pi} \int_0^a \frac{3a^5}{5} \cos^2 \phi \sin \phi \, d\rho d\theta d\phi = \frac{3a^5}{5} \frac{2\pi}{2} \frac{2}{3} \bigg|_0^\pi = \frac{3a^5}{5} \frac{2\pi}{3} \]

7. (a) See book.  
   (b) \( C \) is parametrized by \( x = \cos t, y = \sin t, z = 1, 0 \leq t \leq 2\pi. \)
   
   \[ \int_C \mathbf{F} \cdot d\mathbf{R} = \int_C y \, dx - x \, dy + z \, dz = \int_0^{2\pi} (-\sin^2 t - \cos^2 t) \, dt = -2\pi \]
   
   (c) \[ \int_C \mathbf{F} \cdot d\mathbf{R} = \iint_S \nabla \times \mathbf{F} \cdot \mathbf{n} \, dS = \iint_D -2\mathbf{k} \cdot \frac{dxdy}{\mathbf{n} \cdot \mathbf{k}} = -2 \iint_D dxdy = -2\pi \]

8. See solution to problem 4 on midterm 2 Spring 2000.