Math 20E, Fall **Midterm Exam (Version Green)** **October, 2010**
Instructor: Professor Ni

Name:	_____
Student #:	_____
TA's Name:	_____
Session #:	_____

Instructions

1. **NO CALCULATOR.**
2. **CLOSE BOOK, CLOSE NOTES.**
3. **ID WILL BE CHECKED. GET IT READY!**
4. *Your and your TA'a correct names worth 10pts (bonus). Don't forget.*
5. **SHOW ALL YOUR WORK!**
   (The final answer without steps only earns partial credits.)

Problem	Points
Page 2 (40 points)	
Page 3 (30 points)	
Page 4 (30 points)	
Total (100 points)	

Questions 1 -5 (8pts each) are multiple choices. There may be more than one correct answer. Circle all correct answers to get full credit. Incorrect responses will be penalized accordingly. (For example, if problem X has 2 correct answers and if Y chooses two correct ones and one incorrect (total circle three answers) then he will only receives 4 out of 8 since the incorrect one cancels the credit of one of the two correct choices.)

1. Let $\mathbf{a}$, $\mathbf{b}, \mathbf{c}$ are three vectors with $\cdot$ and $\times$ being the dot and cross product. Which of the following expression(s) is (are) meaningful? *(b, c)*
   (a) $(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$
   (b) $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$
   (c) $\mathbf{a} \times \mathbf{b} + \mathbf{c}$
   (d) $|\mathbf{a}| \times (\mathbf{b} + \mathbf{c})$
   (e) None of them above.

2. Let $\mathbf{a} \neq \mathbf{0}$, $\mathbf{b}, \mathbf{c}$ are three vectors. Which of the following is(are) the correct statement(s)? *(d)*
   (a) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, then $\mathbf{b} = \mathbf{c}$
   (b) If $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, then $\mathbf{b} = \mathbf{c}$.
   (c) If $|\mathbf{a} \cdot \mathbf{b}| = |\mathbf{a} \cdot \mathbf{c}|$ and $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, then $\mathbf{b} = \mathbf{c}$.
   (d) $(\mathbf{a} - \mathbf{b}) \times (\mathbf{a} + \mathbf{b}) = 2(\mathbf{a} \times \mathbf{b})$.
   (e) None of them above.

3. Which of the following function(s) has(have) limit at the origin $\mathbf{0}$? *(c, d)*
   (a) $\frac{x^2-y^2}{x^2+y^2}$
   (b) $\frac{xy^2}{x^2+y^4}$
   (c) $\frac{3x^2y}{x^2+y^2}$
   (d) $\frac{xyz}{x^2+y^2+z^2}$.
   (e) None of them above.

4. Let $\mathbf{a}$, $\mathbf{b}, \mathbf{c}$ are three vectors. Which of the following is(are) correct? *(b, c)*
   (a) $(\mathbf{a} \times \mathbf{b}) \times \mathbf{a} = 0$, for any vectors $\mathbf{a}$ and $\mathbf{b}$
   (b) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} = 0$, for any vectors $\mathbf{a}$ and $\mathbf{b}$
   (c) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = -(\mathbf{b} \times \mathbf{a}) \cdot \mathbf{c}$
   (d) $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times (\mathbf{b} \times \mathbf{c})$
   (e) None of them above.

5. Which following statement(s) is (are) correct? *(b,c,d)*

(a) There exists a function with $f_x = x + 4y$ and $f_y = 3x + y$

(b) $f(x, y) = \ln(x^2 + y^2)$ satisfies $f_{xx} + f_{yy} = 0$.

(c) $f(x, y) = \tan(x + 2010t) + \ln(x - 2010t)$ satisfies $f_{tt} = (2010)^2 f_{xx}$

(d) $f(x, y) = x^{2010} + x^{2008}y^2 + y^{2010}$ satisfies $x^2 f_{xx} + 2xy f_{xy} + y^2 f_{yy} = 2010 \times 2009 f$

(e) None of them above is correct.

For problems 6 and 7, you have to show all related steps. Only correct answer earns limited partial credits.

6. (30 pts) Let α be the plane $4x + 3y + 5z = 50$.

(1) Find the distance between α and the origin.

First the unit normal of the plane is

$$\vec{n} = (3, 4, 5)/5\sqrt{2}$$

The distance is given by taking any point $P = (x_0, y_0, z_0)$ *on the plane* α*, then take* $|(P - 0) \cdot \vec{n}|$

Answer: $5\sqrt{2}$

(2) Find the coordinates of the point (mirror) symmetric to the origin with respect to the plane α.

The line joining the point $(0, 0, 0)$ *perpendicularly to the plane* α *is given by*

$$(4t, 3t, 5t)/5\sqrt{2}.$$

At $t = 5\sqrt{2}$ *it hits the plane and at* $t = 10\sqrt{2}$ *it arrives the mirror image of the origin.*

Answer: $(8, 6, 10)$.

7. (30 pts) Let α be the surface $x^2 + 2y^2 + 3z^2 = 6$. (1) Find the unit outer normal direction at $(1, 1, 1)$. Here unit normal means that it has length 1, and outer normal means that the vector points towards the outside of the region enclosed by the surface. For example the unit outer normal at $(1, 0, 0)$ of the sphere $x^2 + y^2 + z^2 = 1$ is $(1, 0, 0)$.

Trivially the answer is

$(1, 2, 3)/\sqrt{14}$

(2) Assume that at $t = 0$ a particle is ejected from $(1, 1, 1)$ along the outer normal direction with speed 10 units per second. At what time does it cross the sphere $x^2 + y^2 + z^2 = 103$?

The line of the trajectory traveled by the particle is given by

$$(1 + 10t/\sqrt{14}, 1 + 20t/\sqrt{14}, 1 + 30t/\sqrt{14}).$$

Now one just needs to solve t_0 *so that* $(1 + 10t_0/\sqrt{14}, 1 + 20t_0/\sqrt{14}, 1 + 30t_0/\sqrt{14})$ *is on the sphere.*

Answer: $t_0 = (-3 + \sqrt{359})/5\sqrt{14}$.

END OF EXAM