Math 20E Midterm 2, Fall 98, Lindblad.

1. Let $S$ be the triangle with vertices $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$ oriented so that the normal $\mathbf{n}$ points in the direction out from the origin. Let $C$ be the boundary curve of $S$ oriented counterclockwise seen from the side of the normal. Let $\mathbf{F} = (x+y)\mathbf{i} + (y+z)\mathbf{j} + (z+x)\mathbf{k}$.

a) Find the flux of $\mathbf{F}$ out from $S$: $\int_S \mathbf{F} \cdot \mathbf{n} \, dS$.

b) Find the line integral of $\mathbf{F}$ around $C$: $\int_C \mathbf{F} \cdot d\mathbf{R}$

2. Let $R$ be the 3-dimensional region $R = \{x^2/4 + y + z^2/4 \leq 1, y \geq 0\}$. Let $S$ be the surface of $R$ with the normal oriented outwards. Note that $S$ has two parts $\{x^2/4 + y + z^2/4 = 1, y \geq 0\}$ and $\{y = 0, x^2/4 + z^2/4 \leq 1\}$.

a) Find the area of $S$.

b) Find the flux of $\mathbf{F} = x\mathbf{i} - y\mathbf{j} + \mathbf{k}$ through $S$: $\int_S \mathbf{F} \cdot \mathbf{n} \, dS$.

3. Find $\int_R x \, dx \, dy$, where $R$ is the region bounded by $y = \frac{1}{x^2}$, $y = \frac{2}{x^2}$, $y = x$ and $y = 2x$ by making the change of variables $u = x^2y$ and $v = y/x$.

4. Let $S$ be the part of the hyperboloid $x^2 + y^2 - z^2 = 1$ with $0 \leq z \leq 1$. A parametrization of the surface is given by

$$\mathbf{R}(u, v) = (\cos u - v \sin u)\mathbf{i} + (\sin u + v \cos u)\mathbf{j} + v\mathbf{k}, \quad 0 \leq u \leq 2\pi, \quad 0 \leq v \leq 1.$$  

a) Find the area element $dS$ expressed in terms of the parametrization $du \, dv$.

b) Find the surface integral $\int_S z \, dS$. 