

Additional Problems-Set 3

1 Assume that $f(z)$ is holomorphic on the unit disk $D = \{z \mid |z| < 1\}$, satisfying that $|f(z)| \leq 1$. Prove that

$$\frac{|f'(z)|}{1 - |f(z)|^2} \leq \frac{1}{1 - |z|^2}.$$

2 If γ is a piecewise differentiable arc contained in $|z| < 1$ the integral

$$\int_{\gamma} \frac{|dz|}{1 - |z|^2}$$

is called the *hyperbolic length* of γ . For any point z_1 and z_2 in $|z| < 1$, find the arc with the shortest *hyperbolic length* joining z_1 and z_2 . If we call the shortest length of the above arc the *hyperbolic distance* between z_1 and z_2 . Derive a formula for the *hyperbolic distance* $d_H(z_1, z_2)$.