

## Additional Problems-Set 4

**1** Let  $ann(0; r_1, R_1)$  and  $ann(0; r_2, R_2)$  be two annuli. Prove that they are conformally equivalent if and only if  $R_1/r_1 = R_2/r_2$ .

**2** Let  $f(z)$  be an entire function satisfying  $f(z + 1) = f(z)$ . Assume that  $|f(z)| \leq C \exp \alpha |z|$  for some  $\alpha < 2\pi$ . Show that  $f(z)$  must be a constant.

**3** Show that there exists a analytic function defined on  $G = ann(0; 0, 1)$  such that  $f'$  never vanish and  $f(G) = B(0, 1)$ .