Additional Problems-Set 5

1. A positive function $f(x)$ is called log-convex if $\log f(x)$ is convex. Assume that $f(t,x)$ is log convex for every $t \in [0,1]$ and is continuous on t. Prove that $\int_0^1 f(t,x)\,dt$ is log-convex.

2. Prove that for $z \neq 0, 1, 2, \cdots$,

$$\sum_{n=0}^{\infty} \frac{1}{(z+n)^2} = \frac{1}{z} + \frac{1}{2z^2} + \int_0^{\infty} \frac{4\eta z}{(\eta^2+z^2)^2} \frac{d\eta}{e^{2\pi\eta}-1}.$$