

March 12, 2007

Math 140A– Some review problems

Most of these problems are fairly easy. They are intended for review, and will be discussed in class Wednesday and Friday. Unless otherwise stated, here X is a metric space with metric $d(x, y)$ and E is a subset of X .

1. Prove that the image of an interval under a continuous map is again an interval.
2. Any polynomial of odd order has at least one zero.
3. Suppose X is compact and f is a real valued, continuous function on X . Show that either $f(x) = 0$ for some $x \in X$ or there exists $n \in \mathbb{N}$ such that $|f(x)| > 1/n$ for all $x \in X$.
4. Let $x_0 \in X$. Show that the function $x \mapsto d(x, x_0)$ is continuous.

Also: Rudin p. 43: # 23, p. 78: #6(a,b) # 20, #21, #22,

Some brief problems:

(a) Find examples f and g of uniformly continuous real valued functions on such that fg is not uniformly continuous.

(b) Prove that the intersection of any collection of compact subsets of X is compact.

(c) Give examples $\{a_n\}$ and $\{b_n\}$ of sequences of real numbers such that $\liminf a_n$ and $\liminf b_n$ are both finite, but

$$\liminf a_n + \liminf b_n \neq \liminf(a_n + b_n)$$

(d) Show that the following statement is not true in general: If x is a limit point of E then there exists $q \in E$, $q \neq x$, such that for any $r > 0$, $q \in N_r(x)$.

(e) Show that the following statement is not true in general: If f is continuous on a bounded interval (a, b) then for all $\epsilon > 0$ there exists $\delta > 0$ such that

$$|x - y| < \delta \implies |f(x) - f(y)| < \epsilon, \quad \forall x, y \in (a, b).$$