

Monday, November 17, 2008

Math 180a Midterm 2

Instructions No calculators, books, or cell phones may be used. You may have one piece of paper, 8 1/2 x 11, written on one side only. Answer all questions. Explain what you are doing and what your notation means. You may leave your answer in the form of a numerical expression. Please use the notation $\Phi(a)$ for the cumulative distribution function of the standard normal random variable, i.e. $\Phi(a) = (1/\sqrt{2\pi}) \int_{-\infty}^a e^{-x^2/2} dx$. Write clearly!

1. (5 pts.) Let X be a random variable with $\text{Var}(X) = 3$. Find $\text{Var}(1 + 2X)$. (Answer should be a number!)

2. (10 pts.) The average number of calls to a computer help line is 4.5 calls per day. Find the probability that there are fewer than 2 calls on a particular day.

3. (15 pts.) A biased coin falls heads 4 times as often as tails. It is tossed 500 times. Give an expression for the approximate probability (normal approximation) that it lands heads between 250 and 300 times. The answer should be an expression using $\Phi(x)$, with only positive values of x .

4. (15 pts.) Three fair dice are rolled. Let X be the maximum of the three values and Y the sum of the three values. Find $p(3, 6)$, i.e. the probability that $X = 3$ and $Y = 6$. Hint: Be careful!

5. (15 pts.) A bowl of fruit contains 20 apples and 7 peaches. All the apples are fine, but every peach is rotten. Three pieces of fruit are chosen at random. Find the expected number of pieces of rotten fruit that will be chosen.

6. (20 pts.) Let X, Y be jointly continuous random variables given by the joint probability density function

$$f(x, y) = \frac{1}{2}x^2e^{-y}, \quad 0 \leq x \leq y.$$

(a) Find $f_X(x)$, the probability density function of X .

(b) Find $E[X]$, the expected value of X .

You may use without proof the formula $\int_0^\infty x^n e^{-x} dx = n!$

Caution: This is a simplified version of a similar homework problem, but the region on which the function is defined is also simplified. You should draw that region first!

7. (20 pts.) Two mice are to be released from their cages. If the mice are released at times independently and uniformly distributed between 9:00 am and 9:30 am, find the probability that the second one released is let out more than 10 minutes later than the first one.