

Midterm 1 Solutions, Math 180A, Fall 08

October 30, 2008

$$1) \text{ a) } \binom{10}{2, 2, 2, 2, 2}$$

$$\text{b) } \frac{5 \binom{8}{2, 2, 2, 2}}{\binom{10}{2, 2, 2, 2, 2}}$$

$$2) P(\text{no republicans}) = P(\text{all democrats}) = \frac{\binom{50}{3}}{\binom{85}{3}}$$

3) $F = \{\text{system functions}\}$. We are looking for

$$P(C_2|F) = \frac{P(C_2F)}{P(F)} = \frac{P(C_2)}{1 - P(F^c)}$$

$P(F^c) = P\left(\bigcap_{i=1}^5 C_i^c\right) = \prod_{i=1}^5 P(C_i^c) = \prod_{i=1}^5 (1 - p_i)$ by independence of C_i . The desired probability is

$$\frac{p_2}{1 - \prod_{i=1}^5 (1 - p_i)}$$

$$4) P(W) = \frac{n}{n+1}, P(M) = \frac{1}{n+1}$$

$$P(F) = P(F|M)P(M) + P(F|W)P(W) = \frac{n(.55) + (.4)}{n+1}$$

5)

$$\begin{aligned} P(W|HH) &= \frac{P(HH|W)P(W)}{P(HH|W)P(W) + P(HH|R)P(R) + P(HH|B)P(B)} \\ &= \frac{(.3)(.4)^2}{(.3)(.4)^2 + (.3)(.2)^2 + (.4)(.7)^2} \end{aligned}$$

Since by independence $P(HH|W) = P(H|W)^2$, $P(HH|B) = P(H|B)^2$, $P(HH|R) = P(H|R)^2$.