Solutions to Midterm 2, Math 180A, Fall 08

November 30, 2008

1) $Var(1+2X) = 4Var(X) = 12$

2) Poisson $\lambda = 4.5$
$P(X < 2) = P(X = 0) + P(X = 1) = e^{-4.5} + (4.5)e^{-4.5}$

3) $P(H) = 4/5, P(T) = 1/5$. The number of heads on 500 tosses can be approximated by a normal random variable with parameters $\mu = 500 \cdot (4/5) = 400$ and $\sigma = \sqrt{500 \cdot (4/5) \cdot (1/5)} = \sqrt{80}$.

The desired probability is approximately equal to

$P\Big(\frac{249.5 - 400}{\sqrt{80}} < Z < \frac{300.5 - 400}{\sqrt{80}}\Big)$, where Z is the standard normal.

The probability above is

$\Phi\Big(\frac{300.5 - 400}{\sqrt{80}}\Big) - \Phi\Big(\frac{249.5 - 400}{\sqrt{80}}\Big) \approx 0$

4) $\{3, 6\} = \big\{$3 is the maximum, sum is 6$\big\} = \big\{$all permutations of $(1, 2, 3)\big\}$. There are $3! = 6$ such permutations. Every outcome is equally likely with probability $(1/6)^3$. Therefore $P(3, 6) = 6 \cdot (1/6)^3 = 1/36$.

5) Let X denote the number of rotten fruit selected. Then X is a hypergeometric random variable (page 178 of the text) with parameters $n = 3, N = 27, m = 7$.

$$\mathbb{E}[X] = \frac{n \cdot m}{N} = \frac{7}{9}$$

6) a) $f_X(x) = \int_x^\infty f(x,y)dy = \int_x^\infty 1/2x^2 e^{-y} dy = (1/2)x^2 e^{-x}$

b) $\mathbb{E}[X] = \int_0^\infty x f_X(x) dx = \int_0^\infty (1/2)x^3 e^{-x} dx = 3!/2 = 3$ by the hint.

7) Let X, Y be the times of the first and second departure. The joint density of (X, Y) is given be

$$f(x,y) = \frac{1}{30^2} \; 0 \le x, y \le 30$$

We are looking for

$$P(|Y - X| > 10) = 2\int_0^{20}\int_{x+10}^{30} \frac{1}{30^2} dy dx = 4/9$$