

Place all books, papers under your desk. On the front of your blue book write your name and section number. In a column, number one to ten. Graphing calculators and a 3×5 note card is permitted. Turn in the note card with your final exam. Box answers and show all your work. Good luck.

1. (a) 10 pts. Find the equation of a plane passing through $P = (3, 2, 1)$, $Q = (1, 1, 1)$ and $R = (-1, 2, 4)$.
 - (b) 10 pts. Find the equation of a line through $(2, 3, 7)$ parallel to the line of intersection of the planes $2x + y + z = 0$ and $x - y + 7z = 0$.
 - (c) 10 pts. Draw several level curves of $f(x, y) = \frac{x}{y}$. Compute $\vec{\nabla} f(1, 2)$ and sketch the $\vec{\nabla} f(1, 2)$ and the level curve passing through $(1, 2)$.
2. The graph of the surface $z = f(x, y)$ is shown in the following figure.
- (a) 10 pts. Is the directional derivative of f in the direction $\vec{i} + \vec{j}$ positive or negative at $(2, 5)$?
 - (b) 10 pts. Is the directional derivative of f in the direction of $-\vec{i} + \vec{j}$ positive or negative at $(2, 5)$? Give **reasons** for your answer.
3. (a) 15 pts. Let $f(x, y, z) = \frac{x+y}{z}$. Compute the directional derivative of f at $(2, 3, 4)$ in the direction of the vector $\langle 1, -1, 2 \rangle$.
- (b) 30 pts. Find the maximum and minimum values of $f(x, y) = 2x^2 + y^2 - y$ on the region $x^2 + y^2 \leq 1$.
- (c) 10 pts. Two legs of a right triangle are measured as 5 m and 12 m respectively, with a possible error of 0.2 cm in each. Use differentials to estimate the maximum error in the calculation of the length of the hypotenuse.

4. 20 pts. The elevation of a mountain is given by the equation $f(x, y) = 100 - \frac{x^2}{2} - \frac{y^2}{3}$. Suppose we are at the point $(0, 3, 97)$ and we want to walk to the top at a 30° incline. In which direction should we go? (Your answer should be a vector.)

5. (a) 10 pts. Switch the order of integration

$$\int_0^{16} \int_0^{\sqrt{x}} f(x, y) dy dx.$$

- (b) 20 pts. Evaluate $\iint_R (x - 2y) dx dy$ where R is the parallelogram with vertices $(1, -1)$, $(3, 0)$, $(4, 0)$, $(2, -1)$.

6. 20 pts. Find the volume of the solid bounded above by the cone $4z^2 = x^2 + y^2$ and below by $z = 0$ and on the sides by $x^2 + y^2 = 2y$.

7. 20 pts. Find the Maclaurin series for $\frac{1}{(1+x)^2}$. Give your answer in summation notation.

8. (a) 10 pts. Solve the initial value problem $xy' - 3y = x^2$, $x > 0$, $y(1) = 0$.

- (b) 25 pts. A spring with a mass of 2 kg has damping constant 16 and a force of 12.8 N keeps the spring stretch 0.2 m beyond its natural length. Find the position of its mass at a time t if it starts at equilibrium position with a velocity of 2.4 m/second .

- (c) 15 pts. Solve $y'' - 6y' + 34y = 0$.

9. (a) 20 pts. Evaluate $\int_0^1 \int_x^1 \sin(y^2) dx$.

- (b) 20 pts. Find $\iiint_E x dV$ where E is bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$.

10. 20 pts. Find all critical points of $f(x, y) = y^4 + 2xy + x^2$ and classify each as a local maximum, minimum or saddle point.