290 Qual Prep Problem Set 4 — Summer 2007

1. **Intersection Theory.**
   (a) Use intersection theory to compute the cohomology ring $H^*(S^n \times S^m; \mathbb{Z})$.
   (b) Use intersection theory to compute the cohomology ring $H^*(S^1 \times S^1 \times S^1; \mathbb{Z})$.
   (c) Describe submanifold representatives of the generators of the homology groups of $\mathbb{C}P^n$, and explain how to use these to determine the cohomology ring structure.
   (d) Use intersection theory to compute the cohomology ring of a 2-sphere with k handles attached (i.e. a connected sum of k torii).

2. **Standard Qual Questions.**
   (a) Compute $\pi_3(\mathbb{R}P^3 \vee S^3)$.
   (b) Let M be a closed connected simply-connected 4-manifold. Show that $H_1(M; \mathbb{Z}) = H_3(M; \mathbb{Z}) = 0$ and that $H_2(M; \mathbb{Z})$ is a free abelian group.
   (c) Let X be a compact space having the homotopy type of $S^3 \vee S^5$. Determine if X can be a manifold.
   (d) Prove that any continuous map $f : S^2 \times S^2 \to \mathbb{C}P^2$ has even degree.