

Math 161, Spring 2000, Assignment 1
Due April 19, 2000

1. How close is $e^{\pi\sqrt{163}}$ to the closest integer? Give an answer correct to 3 significant digits, in scientific notation.
2. Write named functions identical in effect to the following pure functions.
 - (a) $(\#^3)\&$
 - (b) $(\{\#, E^{(2\#)}\})\&$
 - (c) $(\#1^3 + \#2^2)\&$
3. Write the following functions (displayed in standard mathematical notation) in pure function form.
 - (a) $f(x, y) = \cos(xy) \exp(-2(x^2 + y^2));$
 - (b) $g(x) = 2 \ln x$ for $x > 0$, $g(x) = 1$ for $x \leq 0$.
4. Plot graphs (on the same axes) of (a) the Gamma function $\Gamma(t)$ for $1 \leq t \leq 5$; (b) its Stirling formula approximant, $\sqrt{2\pi}e^{-(t-1)}(t-1)^{t-1/2}$; (c) the ratio of the two. The axes should pass through the true origin, the first function should be colored black, the second red and the third blue. Make appropriate legends to indicate the meanings of the colors. (Hints: check out PlotStyle and Hue, and the options to Plot.)
5. Write a sequence of Mathematica expressions (ie, a program) which, for a given positive integer n ,
 - (a) generates a list of binomial coefficients $\binom{n}{k}$ for $0 \leq k \leq n$ (hint: Table, but be careful about where the list starts);
 - (b) plots the values (use ListPlot);
 - (c) on the same graph but colored red, plots the graph of the function

$$g(x) = \frac{2^n}{\sqrt{2\pi}} \exp(-2(x - n/2)^2/n).$$

(This illustrates essentially the so-called normal approximation curve to the binomial distribution, and is closely connected with the preceding exercise.) Show the results of the plots for $n = 5, 20$.

6. Mathematica is unable to get a general formula for $\int |\sin x| dx$. Write a Mathematica procedure to do this.
7. ParametricPlot3D can be used to plot a surface in 3-space in parametric form, with x, y and z given as functions of a pair of parameters u, v . Use this to make a graph of a hemisphere with plane polar coordinates as the parameters. Do this with a sequence of viewpoint making an entire vertical circuit around the hemisphere, and use this to make an animated view of the surface.