Extra Problems for 1/9/09

1. Assume that $u, v, w \in \mathbb{Z}$, $w > 0$ and $\det \begin{bmatrix} u & v \\ w & 1 \end{bmatrix} = 1$. Show that if d is a positive integer that is not a square of an integer and $\sqrt{d} = [q_0, q_1, q_2, ...]$ is its simple continued fraction then

$$\frac{u\sqrt{d} + v}{w\sqrt{d} + 1} = [v, w, q_0, q_1, ...].$$

2. Calculate the simple continued fraction for $\sqrt{7}$ and for $\frac{3\sqrt{7}+2}{\sqrt{7}+1}$ check your work using pari.

3. Use Pari to calculate the simple continued fraction decomposition for $e = \exp(1)$ with the default precision then set the precision to 50 (\p 50) and 100. What would you guess is the partial fraction decomposition for e? Try the same for π (Pi). Doyou see any pattern?