Extra Problems 3/4/09

1. Clearly \( x^2 - x + 41 \) is not a prime for \( x = 41 \). Show that it is a prime for \( 1 \leq x \leq 40 \).

2. Show that \( \binom{2n}{n} \geq \frac{2^{2n}}{2^n} \) for \( n \geq 4 \). Use this lower bound in place of the one we used \( (2^n) \) to get a better lower bound for \( \pi(x) \).

3. Let \( p_n \) be the \( n^{th} \) prime. Prove that \( p_n \leq 2^n \).