

Extra Homework for 10/27

1. Let $\phi(n)$ be Euler's phi function of n . Let $n > 0$ be an integer. Show that either $\phi(3n)$ is either $3\phi(n)$ or $2\phi(n)$. Determine the n such that $\phi(3n) = 3\phi(n)$. Formulate a general theorem that determines the values of $\phi(pn)$ for p a prime and $n \in \mathbb{Z}_{>0}$.

2. How many solutions modulo 20 are there to the equation $x^2 \equiv x \pmod{20}$?

*3. Prove that $\gcd(2^e - 1, 2^f - 1) = 2^{\gcd(e,f)} - 1$ for $e, f \in \mathbb{Z}_{>0}$.