Practice Midterm Examination Math 100A Fall 2003

This collection of problems is to give you an idea of the type of problems that will be on the actual midterm. All statements must be backed with reasons. You will get partial credit if you can do a part of a problem (for example, in problems 4 and 5 have two parts).

1. Which of the following rules define equivalence relations on $\mathbb{R}$ (the real numbers)? For the cases that are equivalence relations describe the equivalence classes.

a) $s \equiv t$ if $s = t$ or $st = 0$.
b) $s \equiv t$ if $s = t$ or $st = 1$.
c) $s \equiv t$ if $s^2 = t^2$.

2. Consider the following binary operations on $\mathbb{Z}$. Check whether they define a mondoid, a group or neither.

a) $x * y = x + y + 2$ for $x, y \in \mathbb{Z}$.
b) $x * y = xy + 2x + 2y + 2$ for $x, y \in \mathbb{Z}$.
c) $x * y = x^2y^2$ for $x, y \in \mathbb{Z}$.

3. Calculate $(\overline{16})^{27}$ in $\mathbb{Z}_{17}$.

4. Let $G = \mathbb{Z}_8$ under multiplication. Give the Cayley table for G^*. Prove that there exist elements $x, y \in G^*$ such that

$$G^* = \{x^a y^b | a, b \in \{0, 1\}\}.$$

5. Let G a group and let $g, h \in G$ be such that both have order 2 and assume that $gh = hg$. Prove that gh has order 2. Let $g = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, h = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$ in the group of all 2×2 invertible matrices under matrix multiplication.. Show that $g^2 = h^2 = I$ but the order of gh is 6. Why doesn't this contradict the first part of the problem?