

Math 21C Final, Fall 02. Lindblad.

- A particle moves with position vector given by $\mathbf{r}(t) = t^2 \mathbf{i} + (1 - t^2) \mathbf{j} + t^3 \mathbf{k}$.
 - Find the equation of the tangent line to the curve at $t = 1$!
 - What distance does the particle travel between time $t = 0$ and $t = 1$?
- Given the three points $P(0, 1, 1)$, $Q(0, 2, 0)$ and $R(2, 1, 0)$.
 - Find the equation of the plane containing the three points!
 - Find the Area of the triangle with the three points as corners!
 - What are the cosines of the angles at the three corners of the triangle in (b)?
- The temperature of a metal ball centered at the origin is given by $T = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$.
 - Find the rate of change of T at $(1, 2, 2)$ in the direction toward the point $(2, 1, 3)$
 - Explain why for any point (x, y, z) in the ball, the direction of greatest increase in the temperature is toward the center of the ball.
- Find the points on the ellipsoid $4x^2 + y^2 + z^2 = 4$ where the tangent plane is parallel to the plane $x + 2y - z = 0$.
 - Find the equation for the tangent plane at these points.
- Find the critical points of $f(x, y) = x^2 + 5y^2 + 3xy + 8$ and determine if they are local max, min or saddle points.
 - Find the max and min of $f(x, y)$ on the set $g(x, y) = x^2 + y^2 = 4$.
 - Find the absolute max and min of $f(x, y)$ over the set $D = \{(x, y) | g(x, y) \leq 4\}$.
- If you send a package with UPS they charge according to weight as well as size. If $x \geq 0$, $y \geq 0$, $z \geq 0$, are the length of the edges of a rectangular box labeled so x is the smallest and z is the largest then in order that the package should not be considered over sized we must have that $G(x, y, z) = 2x + 2y + z \leq 120$ inches. Find the maximum of the volume $V(x, y, z) = xyz$ for a package that is not over sized.
- Write the iterated integral $\int_0^1 \int_1^{1/y} x^2 e^{-x^2} dx dy$ as a double integral over some unbounded domain D in the x - y plane.
 - Evaluate the integral in (a) by changing the order of integration.
- Find $\iint_R (1 + y) \cos(x^2 + y^2) dA$, where $R = \{(x, y); 1 \leq x^2 + y^2 \leq 4\}$.
- Find the area of the part of the plane $x + 2y + z = 8$ above the region $D = \{(x, y); 0 \leq x \leq 1, 2x^2 \leq y \leq 1 + x^2\}$, i.e. of the surface $S = \{(x, y, z); (x, y) \in D, x + 2y + z = 8\}$.
- Let D be the triangle in the x - y plane with vertices $(0, 0)$, $(0, 4)$ and $(1, 2)$. Write the volume of the solid E , below the plane $z = x - y + 20$ and above the triangle D , as a double integral over D and evaluate it.
(i.e. $E = \{(x, y, z) | 0 \leq z \leq x - y + 20, (x, y) \in D\}$.)