Let $\mathbf{a} = \langle a_1, a_2, a_3\rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3\rangle$ be vectors and θ the angle between them.
Length: $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.
Dot product: $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}|\,|\mathbf{b}| \cos\theta$.
$\mathbf{a}$ is parallel to $\mathbf{b}$ if $\mathbf{a} = c\mathbf{b}$, for some scalar c. $\mathbf{a}$ and $\mathbf{b}$ are perpendicular if $\mathbf{a} \cdot \mathbf{b} = 0$.
Vector projection of $\mathbf{b}$ onto $\mathbf{a}$: $\mathbf{proj_a}(\mathbf{b}) = \text{comp}_{\mathbf{a}}(\mathbf{b})\mathbf{u}$, where $\mathbf{u} = \mathbf{a}/|\mathbf{a}|$ is a unit vector and $\text{comp}_{\mathbf{a}}(\mathbf{b}) = \mathbf{b} \cdot \mathbf{u}$ is the component of $\mathbf{b}$ in the direction of $\mathbf{a}$.
Cross product: $\mathbf{a}\times\mathbf{b} = \langle a_2 b_3 - a_3 b_2\,,\ a_3 b_1 - a_1 b_3\,,\ a_1 b_2 - a_2 b_1\rangle$, if $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k},$$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}|\,|\mathbf{b}| \sin\theta$, $\mathbf{a} \times \mathbf{b}$ is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.
The area of the triangle with $\mathbf{a}$ and $\mathbf{b}$ as two edges is $A = |\mathbf{a} \times \mathbf{b}|/2$.
The volume of the parallelepiped with $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ as three edges is $V = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$.
$\mathbf{a} \times \mathbf{b} = 0$ if $\mathbf{a}$ and $\mathbf{b}$ are parallel. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0$ if $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ lie in the same plane.
Parametric equations of a line: $x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$ where (x_0, y_0, z_0) is on the line and $\langle a, b, c\rangle$ is parallel to it: $\langle x, y, z\rangle = \langle x_0, y_0, z_0\rangle + t\langle a, b, c\rangle$.
Equations of planes $ax + by + cz + d = 0$, where $\mathbf{n} = \langle a, b, c\rangle$ is a normal to the plane, i.e. it is perpendicular to it. If $P_0(x_0, y_0, z_0)$ is a point in the plane this can also be written as $\langle a, b, c\rangle \cdot \langle x - x_0, y - y_0, z - z_0\rangle = a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$.
The distance between a point $P_1(x_1, y_1, z_1)$ and the plane is $D = \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$.

Quadratic surfaces: An Ellipsoid $x^2 + y^2 + 2z^2 = 1$, an Elliptic Paraboloid $z = x^2 + y^2$, a Hyperbolic Paraboloid $z = x^2 - y^2$, a Cone $z^2 = x^2 + y^2$ and a Hyperboloid $z^2 = x^2 + y^2 + 1$.
Vector function $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$.

Derivative: $\mathbf{r}'(t) = \lim_{h\to 0} \dfrac{\mathbf{r}(t+h) - \mathbf{r}(t)}{h} = \langle f'(t), g'(t), h'(t)\rangle$, is **tangent** to the curve $\mathbf{r}(t)$

Arclength: $L = \displaystyle\int_a^b |\mathbf{r}'(t)|\, dt$, where $|\mathbf{r}'(t)| = \sqrt{f'(t)^2 + g'(t)^2 + h'(t)^2}$

Functions of several variables $f(x, y)$ and $F(x, y, z)$.
Level curves $f(x, y) = k$ and **level surfaces** $F(x, y, z) = k$.

Partial derivatives:

$$f_x(x, y) = \frac{\partial f}{\partial x}(x, y) = \lim_{h\to 0} \frac{f(x+h, y) - f(x, y)}{h},$$
$$f_y(x, y) = \frac{\partial f}{\partial y}(x, y) = \lim_{h\to 0} \frac{f(x, y+h) - f(x, y)}{h}.$$

Gradient: $\nabla f(x, y) = \langle f_x(x, y), f_y(x, y)\rangle$, $\nabla F(x, y, z) = \langle F_x(x, y, z), F_y(x, y, z), F_z(x, y, z)\rangle$

Chain Rule case 1: $\dfrac{d}{dt} F(\mathbf{r}(t)) = F_x \dfrac{dx}{dt} + F_y \dfrac{dy}{dt} = \nabla F(\mathbf{r}(t)) \cdot \mathbf{r}'(t)$ $\quad \mathbf{r}(t) = \langle f(t), g(t)\rangle$

Geometrically: The gradient is orthogonal to the tangent line of the level curves.
Directional derivative in the direction of a unit vector $\mathbf{u} = \langle a, b\rangle$

$$D_{\mathbf{u}} f(x, y) = \lim_{h\to 0} \frac{f(x + ha, y + hb) - f(x, y)}{h} = f_x(x, y)a + f_y(x, y)b = \nabla f(x, y) \cdot \mathbf{u}$$

Max rate of change is $|\nabla f|$ which occurs in the direction of ∇f.

Chain Rule case 2: If $z = f(x, y)$ where $x = g(s, t)$ and $y = h(s, t)$ then

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}, \qquad \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

Tangent plane: The tangent plane to the surface $z = f(x, y)$ at a point (x_0, y_0, z_0) :

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0), \qquad z_0 = f(x_0, y_0)$$

The tangent plane to a level surface $F(x, y, z) = k$ at a point (x_0, y_0, z_0) is

$$F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0$$

Geometrically: The gradient is orthogonal to the tangent plane of the level surface.

Differentials: If $z = f(x, y)$ and (dx, dy) are variables then the differential of f is

$$dz = f_x(x, y)\,dx + f_y(x, y)\,dy$$

Linear approximation: With $dx = \triangle x$ and $dy = \triangle y$ we have

$$\triangle z = f(x + \triangle x, y + \triangle y) - f(x, y) \sim dz = f_x(x, y)\,\triangle x + f_y(x, y)\,\triangle y$$

Max-min of $f(x, y)$ is a **critical point:** $f_x(x, y) = f_y(x, y) = 0$.

Second derivative test If (x, y) is a critical point and $D = f_{xx} f_{yy} - f_{xy}^2$. Then it is a local min if $D > 0$ and $f_{xx} > 0$, a local max if $D > 0$ and $f_{xx} < 0$ and saddle point if $D < 0$, i.e. neither max nor min.

Lagrange multipliers. To find the max and min of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$ we find all values of (x, y, z) and λ such that

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z), \qquad \text{and} \qquad g(x, y, z) = k$$

Evaluating $f(x, y, z)$ at all the resulting points gives the max and min.

Double integrals. If $R = \{(x, y);\ a \le x \le b,\ c \le y \le d\}$ then:

$$\iint_R f(x, y)\,dA = \int_a^b \int_c^d f(x, y)\,dy\,dx$$

Region of type I: $D = \{(x, y);\ a \le x \le b,\ g_1(x) \le y \le g_2(x)\}$ then

$$\iint_D f(x, y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\,dy\,dx$$

Polar rectangle $R = \{(r, \theta);\ a \le r \le b,\ \alpha \le \theta \le \beta\}$ then

$$\iint_R f(x, y)\,dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\,r dr\,d\theta$$

Surface area, $z = f(x, y)$, $(x, y) \in D$:

$$\text{Area}(S) = \iint_D \sqrt{1 + f_x(x, y)^2 + f_y(x, y)^2}\ dA.$$

Triple Integrals If $B = \{(x, y, z); a \le x \le b,\ c \le y \le d,\ r \le z \le s\}$ then

$$\iiint_B f(x, y, z)\,dV = \int_r^s \int_c^d \int_a^b f(x, y, z)\,dx\,dy\,dz$$

If region of **type 1**: $E = \{(x, y, z);\ (x, y) \in D,\ u_1(x, y) \le z \le u_2(x, y)\}$then

$$\iiint_E f(x, y, z)\,dV = \iint_D \Big(\int_{u_1(x,y)}^{u_2(x,y)} f(x, y, z)\,dz\Big) dA$$