Lecture 9: Quadratic Surfaces.

The homework solutions are available from soft reserves.
Practice midterms are on the web.
Your midterm exam will contain a sheet of useful formulas. You can see the sheet on the web.
There will be a one hour review session this Sunday in this room, starting 11am.

In this lecture we will learn to identify and sketch some simple types of surfaces called quadratic surfaces.

Quadratic surfaces. A Quadratic surface is a set of (x, y, z) satisfying a quadratic equation, .i.e. it is a zero set of a polynomial of degree two in (x, y, z):

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0 \tag{12.6.3}$$

By translations and rotations one can reduce it to either of the two standard forms:

$$Ax^2 + By^2 + Cz^2 + J = 0, \qquad \text{or} \qquad Ax^2 + By^2 + Iz = 0 \tag{12.6.4}$$

Examples: When the coefficient in front of one of the variables vanishes, the surface obtained is a **cylinder**, that is a surface consisting of all lines that are parallel to a given line and pass through a given plane curve. For example
Circular cylinder: $y^2 + z^2 = 1$.
Parabolic cylinder: $z = y^2$.

Next we consider the
Ellipsoid: $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$. We can make a sketch of this by first sketching the traces of the surface in the planes with one variable constant. In fact we can obtain this ellipsoid by stretching the sphere $x^2 + y^2 + z^2 = 1$ by a factor of 2 in the y-direction and by a factor 3 in the z direction. In general, if we have a picture of the surface S given by

$$F(X, Y, Z) = 0$$

and if $a, b, c > 0$, then we get a picture of the surface S' given by

$$F\left(\frac{x}{a}, \frac{y}{b}, \frac{z}{c}\right) = 0$$

by stretching S by a factor of a in the x-direction, a factor b in the y-direction and a factor c in the z-direction.

Next we consider the
One sheet hyperboloid: $x^2 + y^2 - z^2 = 1$.
Notice that this surface depends on x and y through $x^2 + y^2$, which is the square of the distance to the z-axis. Because of this the surface is a surface of revolution about the z axis. A way to **sketch** such a surface is to first draw the curve which

is the intersection of the surface with the y-z coordinate plane and then think of this curve as rotated around the z-axis, by drawing a few circles which are the intersection of the surface with the coordinate planes parallel to the x-y plane, where $z =$ const..

We can apply the same technique to sketch the
Two sheet hyperboloid: $-x^2 + y^2 - z^2 = 1$.
Here, because the surface depends on x and z through $x^2 + z^2$ we have a surface of revolution about the y-axis.

Circular Cone: $x^2 = y^2 + z^2$.
Here, because the surface depends on y and z through $y^2 + z^2$ we have a surface of revolution about the x axis.

Elliptic Paraboloid: $z = x^2 + y^2$.
Again we have a surface of revolution about the z-axis. You should notice that this surface has traces which are circles (ellipses) and traces which are parabolas.

Hyperbolic Paraboloid: $z = y^2 - x^2$.
This is not a surface of revolution about any of the axes. It looks like a saddle for horseback riding. It has traces which are parabolas and traces which are hyperbolas.