Math 21C Midterm 1, Fall 02. Lindblad.

1. Consider the curve defined by the parametric equations

$$x = t, \qquad y = 2t^{3/2}, \qquad \text{where} \qquad 0 \le t \le 2.$$

(a) Write down the equation for the line that is tangent to the curve when $t = 1$.
(b) Find the arc length of the curve.

2. Consider the curve C given by the parametric equations

$$x = \cos(t), \qquad y = 2 + \sin(t), \qquad \text{where} \qquad 0 \le t \le 2\pi.$$

(a) Find the points where the tangent to the curve is vertical.
(b) Find the surface area obtained by rotating the curve around the x-axis.
(The surface is a donut.)

3. Let $\mathbf{a} = \langle 2, 2, 2 \rangle$ and $\mathbf{b} = \langle 5, c, 5 \rangle$ be vectors, where c is to be determined.

(a) For what value of c is $\mathbf{b} = \langle 5, c, 5 \rangle$ orthogonal to $\mathbf{a} = \langle 2, 2, 2 \rangle$.
(b) For what value of c is $\mathbf{b} = \langle 5, c, 5 \rangle$ parallel to $\mathbf{a} = \langle 2, 2, 2 \rangle$.

4. Let $P(1, 0, -3)$, $Q(0, -2, -4)$ and $R(4, 1, 6)$ be points.

(a) Find the equation of the plane through the points P, Q and R.
(b) Find the area of the triangle with vertices P, Q and R.

5. Consider the plane $x + 2y + z + 4 = 0$ and the point $P_1(1, 0, 1)$.

(a) Find the equation of the line though P_1 that is perpendicular the plane.
(b) Find the point P in the plane that is closest to P_1, i.e such that the distance $|\overrightarrow{PP_1}|$ is the shortest distance between the plane and the point P_1.