Solutions to Math 21C Midterm 1, Fall 02. Lindblad.

1. (a) $dy/dx = y'(t)/x'(t) = 3t^{1/3} = 3$ and $(x, y) = (1, 2)$, when $t = 1$, so the eq. of the tangent line is $x - 1 = 3(y - 2)$.

(b) $L = \displaystyle\int_0^1 \sqrt{1+3t}\, dt = \frac{2}{9}(1+3t)^{3/2}\Big|_0^2 = \frac{2}{9}\big(7^{3/2} - 1\big).$

2. (a) $x'(t) = -\sin t = 0$, when $t = 0$ or $t = \pi$.

$$(b) \quad A = \int_\alpha^\beta 2\pi y \sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt = \int_0^{2\pi} 2\pi(1+\sin t)\sqrt{\sin^2 t + \cos^2 t}\, dt$$
$$= \int_0^{2\pi} 2\pi(1+\sin t)\, dt = 2\pi\big(t - \cos t\big)\Big|_0^{2\pi} = 4\pi^2$$

3. (a) $\mathbf{a}\cdot\mathbf{b} = 10 + 2c + 10 = 0$, if $c = -10$ so they are perpendicular then.

(b) paralllel means that $\mathbf{a} = \lambda\mathbf{b}$ for some constant λ. This mean that we must have $2 = \lambda 5$, $2 = \lambda c$ and $2 = \lambda 5$. From the first and last equations it follows that $\lambda = 2/5$ and plugging this in to the second gives that $c = 5$.

4. (a) Let $\mathbf{a} = \overrightarrow{PQ} = \langle -1, -2, -1\rangle$ and $\mathbf{b} = \overrightarrow{PR} = \langle 3, 1, 9\rangle$. A normal to the plane is

$$\mathbf{n} = \mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -2 & -1 \\ 3 & 1 & 9 \end{vmatrix} = (-18+1)\mathbf{i} - (-9+3)\mathbf{j} + (-1+6)\mathbf{k} = -17\mathbf{i} + 6\mathbf{k} + 5\mathbf{k}$$

Since e.g. P is in the plane, the equation is $-17(x-1) + 6y + 5(z+3)$.

(b) The area is $A = |\mathbf{a}\times\mathbf{b}|/2 = \sqrt{17^2 + 6^2 + 5^2}/2$.

5. (a) The normal to the plane is $\mathbf{n} = \langle 1, 2, 1\rangle$ which is parallel to the line and $P_1(1, 0, 1)$ is on the line so the parametric equations of the line are: $x = 1 + t$, $y = 2t$ and $z = 1 + t$.

(b) The shortest distance between the point and the plane is in fact along the line in (a) since that line is perpendicualr to the plane. Therefore we plug in the parametric equations of the line in the equation of the plane to obtain: $1+t+4t+1+t+4 = 0$, i.e. $t = -1$. Plugging this back into the parametric equations gives $(x, y, z) = (0, -2, 0)$.