Math 21C Solutions to Midterm #1 (2/4/02) Version 1 Miatello

(1) When $\theta = \pi/6$ the slope of the tangent line is

$$\frac{dy}{dx} = \frac{y'(\theta)}{x'(\theta)} = \frac{r'(\theta)\sin\theta + r(\theta)\cos\theta}{r'(\theta)\cos\theta - r(\theta)\sin\theta} = \frac{3\cos(3\theta)\sin\theta + \sin(3\theta)\cos\theta}{3\cos(3\theta)\cos\theta - \sin(3\theta)\sin\theta} = \frac{\cos(\pi/6)}{-\sin(\pi/6)}$$

(2) $$L = \int_0^{\pi/4} \sqrt{x'(\theta)^2 + y'(\theta)^2}\, d\theta = ... = \int_0^{\pi/4} \sqrt{r'(\theta)^2 + r(\theta)^2}\, d\theta$$
$$= \int_0^{\pi/4} \sqrt{9\sin^2\theta + 9\cos^2\theta}\, d\theta = \int_0^{\pi/4} \sqrt{9}\, d\theta = 3\pi/4 \quad (1)$$

(3) Let $P_0(2,1,4)$, $P_1(4,3,10)$ and $\mathbf{a} = \overrightarrow{P_0P_1} = \langle 2,2,6\rangle$. The diameter is $|\overrightarrow{P_0P_1}| = \sqrt{2^2+2^2+6^2} = \sqrt{44} = 2\sqrt{11}$ and the center is at the point $P_2 = P_0 + \mathbf{a}/2 = (3,2,7)$ so the equation of the sphere is $(x-3)^2 + (y-2)^2 + (z-7)^2 = 11$.

(4) Let $\mathbf{a} = \overrightarrow{PQ} = \langle -1,-2,-3\rangle$ and and $\mathbf{b} = \overrightarrow{RS} = \langle 1,0,1\rangle$
Then $\mathbf{a}\cdot\mathbf{b} = (-1)1 + (-2)0 + (-3)1 = -4 \neq 0$ so they are not perpendicular.

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -2 & -3 \\ 1 & 0 & 1 \end{vmatrix} = ... = -2\mathbf{i} - (-1+3)\mathbf{j} + (+2)\mathbf{k} = -2\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \neq 0$$

so they are not parallel.

(5) Let $\mathbf{a} = \overrightarrow{PQ} = \langle 1,2,-1\rangle$ and $\mathbf{b} = \overrightarrow{PR} = \langle 2,3,1\rangle$. Then

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -1 \\ 2 & 3 & 1 \end{vmatrix} = ... = (2+3)\mathbf{i} - (1+2)\mathbf{j} + (3-4)\mathbf{k} = 5\mathbf{i} - 3\mathbf{j} + 1\mathbf{k}$$

The area is $A = |\mathbf{a}\times\mathbf{b}|/2 = \sqrt{5^2+3^2+1^2}/2 = \sqrt{35}/2$.