

Math 21C Midterm II, Fall 02. Lindblad.

1. A particle moves with position vector given by $\mathbf{r}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + t^{3/2} \mathbf{k}$.
 - (a) Find the velocity and speed of the particle at time $t = \pi$.
 - (b) How far does the particle travel between time $t = 0$ and $t = 10$?

2. Let $F(x, y) = x^3 - x^2 + y^2 - y + 1$
 - (a) Find the gradient $\nabla F(x, y)$.
 - (b) Find the directional derivative in the direction of $\langle 1, 2 \rangle$ at the point $(2, 3)$.
 - (c) Let $\mathbf{r}(t) = \langle f(t), g(t) \rangle$ be a curve such that $\mathbf{r}(0) = \langle 1, 0 \rangle$ and $\mathbf{r}'(0) = \langle 1, 1 \rangle$. Let $h(t) = F(\mathbf{r}(t))$. Find $h'(0)$.

3. Consider the surface given by $z = f(x, y) = \sqrt{x^2 + 3y^2}$.
 - a) Find the tangent plane to the surface at the point $(1, 1, 2)$.
 - b) A student was asked to find an approximation for $f(1.1, 1.2)$ but the professor did not allow calculators. The student noticed that $f(1.1, 1.2)$ is approximately $f(1, 1) = \sqrt{1 + 3} = 2$. Use the linear approximation to get a better approximation.

4. Let $f(x, y) = x^3 - x^2 + y^2 - y + 1$. Find the critical points of $f(x, y)$ and determine if they are local max, min or saddle points. Are there any absolute max or min?

5. Use Lagrange multipliers to find the point on the hyperboloid $\{(x, y, z); z^2 = x^2 + y^2 + 1, z \geq 0\}$ that is closest to the point $(0, 0, -2)$.