Simplified version of Math 20C Midterm 2/20, 96. Okikiolu.

1. (10p) The dimensions of a closed rectangular box are measured as 20 cm, 10 cm and 5 cm respectively, with a possible error of 0.2 cm in each dimension. Use differentials to estimate the maximum error in calculating the volume of the box.

2. (10p) Suppose that $f(x, y)$ is a function of two variables and that the directional derivative of f in the direction $\mathbf{u} = \langle 1, 0 \rangle$ at the point $(2, 3)$ is 5 i.e. $D_{\mathbf{u}}f(2, 3) = 5$, and that $D_{\mathbf{v}}f(2, 3) = -2$, if $\mathbf{v} = \langle 0, 1 \rangle$.
(a) What is the gradient of f at $(2, 3)$, i.e. $\nabla f(2, 3)$?
(b) In which direction $\mathbf{w}$ is the directional derivative $D_{\mathbf{w}}f(2, 3) = 0$?

3. (15p) Let $F(x, y, z) = z - y^2 + x^2 + 3$.
Find the tangent plane to the level surface $F(x, y, z) = 0$ at the point $(1, 2, 0)$.

4. (25p) Let $f(x, y) = 3x^2 + 12y^2 + 4xy$.
(a) Find the critical points of $f(x, y)$ and determine if they are local max, min or saddle points.
(b) Use Lagrange multipliers to find the max and min of $f(x, y)$ subject to the constraint $g(x, y) = x^2 + 4y^2 = 8$.
(c) Find the absolute max and min of $f(x, y)$ over the set $D = \{(x, y) | g(x, y) \leq 8\}$.

5. (20p) Use Lagrange multipliers to find the point on the surface
$F(x, y, z) = z - y^2 + x^2 + 3 = 0$ with shortest distance to the origin.
(Hint: Minimize the distance squared rather than the distance.)