Math 194, Winter 2008

Homework 4 — Due Wednesday, January 30

1. Consider the Multi-period Binomial Model (the CCR model) with parameter values as in Exercise 2 on page 28 of the text. Under a *forward contract*, the holder of a long position in the contract will buy 100 shares of stock at time $T = 2$ for a fixed (total) price $\$F$. What should the value of F be so that neither the holder of the long position in the contract, nor the holder of the short position (the seller) has an arbitrage opportunity? Identity the relevant European contingent claim, and derive F using arbitrage-free pricing. (Reminder: No money changes hands at time $t = 0$ when the forward contract is written.)

2. Exercise 3, Section 2.4, but skip part (d) (page 29 of the text).

3. Consider the probability space $(\Omega, \mathcal{F}, \mathbf{P})$ in which (i) $\Omega = \{1, 2, 3, 4\}$, (ii) $\mathcal{F}$ is the σ-field of all subsets of Ω, and (iii) $\mathbf{P}[A] = \text{card}(A)/4$ for each $A \in \mathcal{F}$. (Here, card(A), the cardinality of A, is simply the number of elements of A.) This probability space is a model for an experiment with four equally likely outcomes. Let $\mathcal{G}$ be the σ-field generated by the partition $\mathcal{P} = \{\{1, 2\}, \{3, 4\}\}$ and let $\mathcal{H}$ be the σ-field generated by the partition $\mathcal{Q} = \{\{1\}, \{2\}, \{3, 4\}\}$. Notice that $\mathcal{G} \subset \mathcal{H}$. Let X be the random variable defined by $X(i) = i^2$ for $i = 1, 2, 3, 4$.

(a) Compute $\mathbf{E}[X]$.
(b) Compute $\mathbf{E}[X|\mathcal{G}]$.
(c) Compute $\mathbf{E}[X|\mathcal{H}]$.
(d) Compute $\mathbf{E}\left[\mathbf{E}[X|\mathcal{H}]\,|\mathcal{G}\right]$.
(e) Does your answer in (d) confirm the Tower Property of conditional expectations?

4. Use an Excel spreadsheet to compute the arbitrage-free price of a European *put* option with strike price $K = \$130$, based on the Mutlti-period Binomial Model (the CCR model) with the following parameter values: $T = 5$, $S_0 = \$100$, $u = 2$, $d = .6$, $r = 0.1$. (There is a handout available on the same page as the course text that will be helpful for this problem.)