Math 20C

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Answers to even-numbered exercises from sections 14.4-14.7

§**14.4**

(4) 1.3

(20) $(4/5, \ln(25/4)) \approx (.8, 1.83)$

(22) $f(-2.1, 3.1) \approx 4$

§**14.5**

(26) $D_{\langle 12/13,-5/13\rangle} f(2,2) = \langle 2,-2\rangle \cdot \langle 12/13,-5/13\rangle = 34/13$

(34) Increasing, because $D_{\mathbf{v}} f(P) = 12 > 0$.

(40) $(4/\sqrt{17}, 9/\sqrt{17}, -2/\sqrt{17})$ and $(-4/\sqrt{17}, -9/\sqrt{17}, 2/\sqrt{17})$

§**14.7**

(32) (a) There is one critical point at $(1,1)$, and $f(1,1) = 1$.

(b) Minimum: 0 at $(0,0)$; maximum 4 at $(0,2)$

(c) Top edge: minimum 0 at $(2,2)$; maximum 4 at $(0,2)$. Left edge: minimum 0 at $(0,0)$; maximum 4 at $(0,2)$. Right edge: minimum 0 at $(2,2)$; maximum 4 at $(2,0)$.

(d) The maximum value of f is 4 (and it occurs at $(0,2)$ and $(2,0)$); the minimum value of f is 0 (and it occurs at $(0,0)$ and $(2,2)$).

(38) The interior critical points are $(0,0), (0,1), (0,-1), (1,0), (-1,0)$. The values of f at these points are 0, $4e^{-1}$, $4e^{-1}$, $-e^{-1}$, $-e^{-1}$, respectively. On the boundary circle $x^2 + y^2 = 2$ we have $f(x,y) = (5y^2 - 2)e^{-2}$. (And on the boundary y varies from $-\sqrt{2}$ to $\sqrt{2}$.) The maximum value of f on the boundary is therefore $f(0, \pm\sqrt{2}) = 8e^{-2} \approx 1.08$; the minimum value of f on the boundary is $f(\pm\sqrt{2}, 0) = -2e^{-2} \approx -0.27$. Since $4e^{-1} \approx 1.47$ and $-e^{-1} \approx -0.37$, the global maximum of f is $4e^{-1}$ (occurring at $(0,\pm 1)$) and the minimum value is $-e^{-1}$ (occurring at $(\pm 1, 0)$).

(42) The Lagrange multiplier equations for this problem are

$$2(y+z) = \lambda yz, \qquad 2(x+z) = \lambda xz, \qquad 2(x+y) = \lambda xy.$$

Elimination of λ leads to

$$\frac{1}{y} + \frac{1}{z} = \frac{1}{x} + \frac{1}{z} = \frac{1}{x} + \frac{1}{y}.$$

From this it should be clear that $x = y = z = V^{1/3}$.