Math 194, Winter 2020

Homework 2 — Due Tuesday, January 22

1. Consider a single-period binomial model with $r = 1/3$, $B_0 = 1$, $S_0 = 2$, $d = 5/4$, $u = 3/2$, and $p = 1/2$. (You can take the sample space Ω to be $\{\omega_1, \omega_2\}$, with ω_1 corresponding to the stock price going "up", and ω_2 corresponding to the stock price going "down".)

(a) Compute B_1.

(b) Compute $S_1(\omega_1)$ and $S_1(\omega_2)$, and the probability of each outcome.

(c) For the trading strategy $\psi = (2, 4)$, compute $V_0(\psi)$, $V_1(\psi)(\omega_1)$, and $V_1(\psi)(\omega_2)$.

(d) Let X be a European call option with strike price \$2.50 and expiration time $T = 1$.

- (i) Find $X(\omega_1)$ and $X(\omega_2)$.
- (ii) Find the replicating strategy $\phi = (\alpha_1, \beta_1)$ for X
- (iii) Find the manufacturing cost for that strategy. That is, compute $V_0(\phi)$.

(e) Give an example of arbitrage opportunity if the claim X can be purchased for $C_0 = 1/16$ (dollars) at time 0.

2. Repeat the steps of Exercise 1, with the following data: $r = 1/4$, $B_0 = 1$, $S_0 = 3$, $d = 1$, $u = 2$, and $p = 3/4$. Use the trading strategy $\psi = (3, -2)$ in part (c) . The contingent claim X is now a European *put* option with strike price $K = \$4$. Use $C_0 = 1$ in doing part (e).

3. [Exercise 1 in section 2.4 of the text (page 28).] Consider a single period CRR model with $S_0 = \$100$, $S_1 = \$200$ or $\$50$, $r = 0.25$.

(a) Find the arbitrage free price of a European call option for one share of stock where the strike price is $K = \$100$ and the exercise time $T = 1$.

(b) Find a hedging strategy that replicates the value of the option described in (a).

(c) Suppose the option in (a) is initially priced at \$1 above the arbitrage free price. Describe a strategy (for trading in stock, bond and the option) that is an arbitrage.

(d) What is the arbitrage free initial price for a put option with the same strike price and exercise time as the call option described in (a)?