Math 194, Winter 2020

Homework 9 — Not to be handed in

1. Consider the sample space $\Omega = \{1, 2, 3\}$ and the probability measure $\mathbf{P}$ giving each point weight $1/3$. On this probability space consider the finite model with $T = 1$, $S_0^0 = S_1^0 = 1$, $S_0^1 = 2$, and

$$S_1^1(1) = 1, \qquad S_1^1(2) = 2, \qquad S_1^1(3) = 4.$$

(a) This market is viable, so there is at least one EMM. Find all possible EMMs.

(b) Compute $V_+(X)$ and $V_-(X)$ for the contingent claim X given by

$$X(1) = 1, \qquad X(2) = 3, \qquad X(3) = 9.$$

(c) Find a trading strategy $\phi = (\phi_1^0, \phi_1^1)$ such that $V_1(\phi) \geq X$ and $V_0(\phi) = V_+(X)$.

(d) Consider the contingent claim Y given by

$$Y(1) = 2, \qquad Y(2) = 4, \qquad Y(3) = 8.$$

Without actually finding a hedging strategy, show that Y can be replicated. [Hint: Use Corollary 3.5.2 on page 49 of the text.]

2. For this problem we use the finite model presented in Exercise 3 (Section 3.7, page 53 of the text), but with $r = 0$.

(a) This model is viable but not complete. Describe all possible EMMs.

(b) Consider the European call option $X = (S_2 - 5)^+$. Compute $V_+(X)$ and $V_-(X)$.

(c) Is X replicable? [Hint: The answer from (b) will be helpful.]

3. For this problem we use the finite model presented in Exercise 2 (Section 3.7, pages 52–53 of the text).

(a) Explain why this model is viable and complete.

(b) Find the no-arbitrage price for the European put option $X = (5 - S_2^1)^+$.

(c) Consider the random variable $Y = [S_2^1]^2$ (the square of the stock price at time $t = 2$). Compute $\mathbf{E}^*[Y|\mathcal{F}_1]$ and $\mathbf{E}^*[Y|\mathcal{F}_0]$.