Math 180B, Winter 2021

Homework 1, due January 11

1. A fair coin is tossed 20 times. Let X be the number of heads thrown in the first 10 tosses, and let Y be the number of heads tossed in the last 10 tosses. Find the conditional probability that $X = 6$, given that $X + Y = 10$.

2. Let X_1 and X_2 be independent Poisson random variables with parameters λ_1 and λ_2. Show that for every $n \geq 1$, the conditional distribution of X_1, given $X_1 + X_2 = n$, is binomial, and find the parameters of this binomial distribution.

3. An item is selected uniformly at random from a collection labeled $1, 2, \ldots, n$. Denote its label by X. Now select an integer Y uniformly at random from $\{1, \ldots, X\}$. Find:

(a) $\mathbf{E}(Y)$;

(b) $\mathbf{E}(Y^2)$;

(c) $\mathrm{Var}(Y)$;

(d) $\mathbf{P}(X + Y = 3)$.

4. Let X, Y, and Z be independent random variables, each with the standard normal distribution. Compute the following:

(a) $\mathbf{P}[X + Y > Z + 2]$;

(b) $\mathrm{Var}[3X + 4Y]$;

(c) $\mathbf{P}[3X + 4Y < 5]$.

[You can download a table of the normal cdf at the Math 180B website.]

In addition, do the following problems from the textbook *An Introduction to Stochastic Modeling* (Fourth Edition) by Pinsky and Karlin:

Pages 51–52:. Exercise 2.1.2, Problem 2.1.6