Day 01 – The Pigeon-Hole Principle

**Theorem 1** (Pigeon-Hole Principle). Let \( n \) and \( k \) be positive integers with \( n > k \). Suppose we want to distribute \( n \) identical balls into \( k \) identical boxes. Then there will be at least one box in which we place at least two balls.

**Theorem 2** (Generalized Pigeon-Hole Principle). Let \( n, m \) and \( k \) be positive integers with \( n > km \). Suppose we want to distribute \( n \) identical balls into \( k \) identical boxes. Then there will be at least one box in which we place at least \( m+1 \) balls.
Example 1. At a party with 100 people, there will be two (distinct) people with the same number of friends.
Example 2. Given 17 points inside an equilateral triangle of side length one. Prove that there are two points among them whose distance is no more than $1/4$.

You try: Assume that 101 distinct points are placed in a $10 \times 10$ square such that no three of them lie on a line. Prove that there exist three points that form a triangle whose area is at most one.
**Definition:** Let $m, n$ be integers. We say “$m$ divides $n$” or “$n$ is divisible by $m$” if there exist an integer $r$ such that $n = m \cdot r$.

We write $m|n$ to denote that $m$ divides $n$.

**Example 3.** Prove that there exists a positive integer $n$ of the form $7777 \cdots 7$ such that $2017|n$. 


Example 4. Given a sequence $S$ of $n$ integers $a_1, a_2, \ldots, a_n$. Prove that there exists a consecutive subsequence of $S$ whose sum is divisible by $n$. 
**Example 5.** Prove that if we place more than 9 kings on an $8 \times 8$ chessboard then at least two of them will **attack the same square**.